

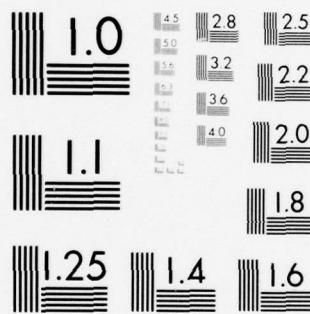
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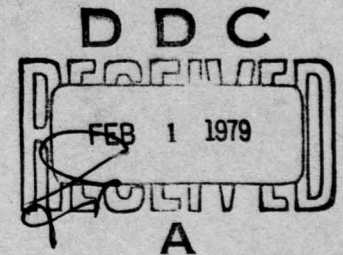
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A METHOD FOR CALCULATING WAVE PACKET TRAJECTORIES AND WAVE HEIGHTS

by J. Ernest Breeding, Jr.
K.C. Matson
Nourollah Riahi

TECHNICAL REPORT No. JEB-1
Department of Oceanography
Florida State University

MARCH, 1978



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State University, Tallahassee, March 1978.

There is a miscopy in Equation (2-48) on page 13.
The equation should read

$$\begin{aligned} \frac{\partial^2 U}{(\partial x')^2} = & \frac{\cosh I}{2} \left\{ - \left[(\sinh I + I) \frac{\partial v}{\partial x'} + v \frac{\partial I}{\partial x'} (1 - I \coth I) \right] \coth I \frac{\partial I}{\partial x'} + \right. \\ & + (\sinh I + I) \frac{\partial^2 v}{(\partial x')^2} + \frac{\partial v}{\partial x'} \frac{\partial I}{\partial x'} (2 + \cosh I - I \coth I) + \\ & \left. + v \left[\frac{\partial^2 I}{(\partial x')^2} (1 - I \coth I) + \left(\frac{\partial I}{\partial x'} \right)^2 \cosh I (I \cosh I - \cosh I) \right] \right\} \quad (2-48) \end{aligned}$$

The FORMAT statements at MAIN 122 and MAIN 124 on
page 39 should be changed to

1 FORMAT (6F10.7)

2 FORMAT (12F2.0)

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TECHNICAL REPORT NO. JEB - 1
Department of Oceanography
Florida State University

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A METHOD FOR CALCULATING WAVE PACKET
TRAJECTORIES AND WAVE HEIGHTS
Part 1.

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ABSTRACT

The theory and numerical methods are presented for determining the paths of gravity water wave packets. A ray curvature expression is used to determine the wave packet trajectories where the speed of the packet is given by $G = (d\omega/dk) \cos \phi$. The symbol ω denotes the angular frequency, k is the wave number, and ϕ is the difference between the direction of the wave packet and the direction of the wavelets within the packet. At each point of the wave packet trajectory the wavelet direction is determined using Snell's law with phase velocity. The wave height is computed along the wave packet paths accounting for the effects of shoaling, refraction, and energy dissipation. The computer program is described and sample printouts and plots are presented.

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TABLE OF CONTENTS

	<u>Page</u>
Abstract	ii
Acknowledgments	iii
Table of Contents	iv
List of Figures	vii
Chapter I	1
Chapter II THEORY	3
2.1 Ray Curvature for Wave Packets	3
a. Locally parallel wave speed contours	3
b. Properties of the group ray curvature	4
c. Wavelet directions	5
2.2 Spatial Derivatives of G , U , v , and h	7
a. Determination of h and its partial derivatives	7
b. Rotation of axes to make computations	9
c. Derivatives of v	11
d. Derivatives of u	13
e. Derivatives of G	13
2.3 Computation of Gravity Water Wave Packet	14
a. Determining the path	14
b. Reflection points	15
2.4 Wave Height Calculations	16
a. Without energy dissipation	17
b. With energy dissipation	18

	<u>Page</u>
2.5 Refraction Coefficient	18
a. Ray separation equation	19
b. Solution for parallel water depth contours	19
c. Numerical solutions of the ray separation equation	20
d. Reflection points	23
e. Caustics and focal points	23
2.6 Friction Coefficient	24
2.7 Wave Breaking Criterion	24
Chapter III THE COMPUTER PROGRAM	25
3.1 Description of the Computer Program	25
3.2 Program MAIN	26
3.3 Subroutine TITLE	26
3.4 Subroutine AXIS2	26
3.5 Subroutine NUMCON	26
3.6 Subroutine SHORE	27
3.7 Subroutine RAYN	27
3.8 Subroutine MOVE	29
3.9 Subroutine HEIGHT	32
3.10 Subroutine SURFCE	35
3.11 Subroutine VELCTY	35
3.12 Subroutine CONDER	36
3.13 Subroutine PCD	36
3.14 Subroutine STORE	36
3.15 Subroutine DRAW	36
3.16 Listing of the Computer Program	37

	<u>Page</u>
Chapter IV USING THE COMPUTER PROGRAM	58
4.1 Preparation of the Water Depth Grid	58
4.2 Preparing a Computer Run	59
4.3 The Printed Output when $NPT \neq 0$	62
4.4 The Printed Output when $NPT = 0$	65
4.5 The Plots	66
4.6 Examples of Computer Output	66
References	81
Notation	83
Distribution List	96

LIST OF FIGURES

	<u>Page</u>
Figure (2-1). Relationship between the Wave Packet and Wavelet Incremental Distances	6
Figure (2-2). Selection of Water Depths about Ray Point (X,Y) = (4.2, 3.8)	8
Figure (2-3). Relationships between the Coordinate Systems, a Ray, and a Depth Contour ...	10
Figure (4-1). Format of the Input Parameters	60
Figure (4-2). Sample Input Data for a Computer Run	63
Figure (4-3). Listing for Ray Number 1 of Sample Input Data	68
Figure (4-4). Listing for Ray Number 2 of Sample Input Data	69
Figure (4-5). Listing for Ray Number 3 of Sample Input Data	70
Figure (4-6). Listing for Ray Number 4 of Sample Input Data	71
Figure (4-7). Listing for Ray Number 5 of Sample Input Data	72
Figure (4-8). Listing for Ray Number 6 of Sample Input Data	73
Figure (4-9). Plot for Sample Input Data	74
Figure (4-10). Gulf of Mexico off the Southwestern Florida Coast	75
Figure (4-11). Plot of Rays off the Southwestern Florida Coast	76
Figure (4-12). Listing of Rays when NPT = 0	77
Figure (4-13). Listing of Rays Moving toward Deep Water	78

	<u>Page</u>
Figure (4-14). Listing of Ray Particulars Near a Reflection Point	79
Figure (4-15). Listing for a Ray with a False Reflection	80

CHAPTER I

It has been shown by Breeding (1978) that a wave packet refracts according to Snell's law with the geometric group velocity G where

$$G = (d\omega/dk) \cos \phi \quad (1-1)$$

The symbol ω denotes the angular frequency and k is the wave number. The angle ϕ is the difference between the direction of the wave packet and the direction of the wavelets within the packet. The wavelet direction at each point of the wave packet trajectory is determined by Snell's law with phase velocity.

In this work a numerical method is presented for determining the paths of gravity water wave packets. Further, a procedure is developed for computing the wave heights along the paths accounting for the effects of shoaling, refraction, and energy dissipation.

There are a number of papers in which numerical methods are presented for calculating and plotting the trajectories of monochromatic rays. Skovgaard, et al (1975) summarize a number of these methods and present one of their own. The numerical methods for calculating and plotting rays which are presented in this work are based on the Wilson (1966) program. However, extensive modifications of the Wilson program were required in order to compute the path of a wave packet.

In Chapter II, an expression is derived for the ray curvature of a wave packet. The assumption is made that the wave speed contours are locally parallel in the vicinity of each point of a ray. Properties of the packet ray curvature are discussed, and a procedure is described for computing the wavelet directions. A method is described for computing the water depth, phase speed, group speeds, and their spatial derivatives from a square grid of water depth values. To simplify the ray curvature calculations a coordinate system is defined with an axis parallel to the water depth contours at each ray point. The numerical procedure for computing the trajectories of gravity water wave packets is described. Rules for dealing with reflection points, which occur when the ray curvature becomes infinite, are established.

The shoaling, refraction, and friction coefficients used to compute the wave height are presented. The refraction coefficient is evaluated on the assumption that the wave speed contours are locally parallel in the vicinity of a ray point. A method for computing the refraction coefficient near a

reflection point is described. A procedure for locating caustics and focal points is discussed. Chapter II ends with a discussion of the wave breaking criterion.

Chapter III contains a description of the computer program and a program listing. A guide to using the program and an explanation of the computer output is given in Chapter IV. The principal notation used in the report is presented following the references.

CHAPTER II THEORY

2.1 Ray Curvature for Wave Packets. The ray curvature κ_v of a ray moving with phase speed v was derived by Munk and Arthur (1952) and Arthur, et al (1952) as

$$\kappa_v = \frac{d\gamma}{ds_v} = \frac{1}{v} \left(\sin \gamma \frac{\partial v}{\partial x} - \cos \gamma \frac{\partial v}{\partial y} \right) \quad (2-1)$$

where x, y are the Cartesian coordinates, γ is the direction of the ray with respect to the positive x -axis, and s_v is the arc length along the ray.

The ray curvature κ_G for the trajectory of a wave packet is given by

$$\kappa_G = \frac{d\theta}{ds_G} = \frac{1}{G} \left(\sin \theta \frac{\partial G}{\partial x} - \cos \theta \frac{\partial G}{\partial y} \right) \quad (2-2)$$

where θ is the direction of the ray with respect to the positive x -axis, ds_G is an element of arc length along the ray, and G is the geometric group speed defined by (Breeding, 1978)

$$G = U \cos \phi \quad (2-3)$$

where

$$U = \frac{\partial \omega}{\partial k} \quad (2-4)$$

is the collinear group speed, $\omega = 2\pi f$ is the angular frequency, $f = 1/T$ is the frequency, T is the wave period, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, and

$$\phi = \theta - \gamma \quad (2-5)$$

a. Locally parallel wave speed contours

The calculations are simplified by making the assumption that the wave speed contours are locally parallel in the vicinity of each point of the ray trajectory. Further, we chose a $x'y'$ -coordinate system such that the y' -derivatives of v , U , and G are zero. Accordingly, in the primed system the space derivative of Equation (2-3) is

$$\frac{\partial G}{\partial x'} = \frac{\partial U}{\partial x'} \cos \phi' - U \sin \phi' \left(\frac{\partial \theta'}{\partial x'} - \frac{\partial \gamma'}{\partial x'} \right) \quad (2-6)$$

Expressions for the space derivatives of θ' and γ' are derived from Equations (2-1) and (2-2) where

$$\partial x' = \partial \lambda_v \cos \gamma' \quad (2-7)$$

$$\partial x' = \partial \lambda_G \cos \theta' \quad (2-8)$$

Thus,

$$\frac{\partial \gamma}{\partial x'} = \frac{\tan \gamma'}{v} \frac{\partial v}{\partial x'} \quad (2-9)$$

$$\frac{\partial \theta}{\partial x'} = \frac{\tan \theta'}{G} \frac{\partial G}{\partial x'} \quad (2-10)$$

where it should be noted that $dy = dy'$, and $d\theta = d\theta'$, $\phi = \phi'$, and $\kappa_G = \kappa_G'$. When Equations (2-9) and (2-10) are substituted into (2-6) and the terms rearranged, it is found that

$$\frac{\partial G}{\partial x'} = \frac{\frac{\partial U}{\partial x'} \cos \phi + \frac{U \sin \phi \tan \gamma'}{v} \frac{\partial v}{\partial x'}}{1 + \tan \phi \tan \theta'} \quad (2-11)$$

After Equations (2-2) and (2-11) are combined and the result is simplified, it is found that

$$\kappa_G = \frac{\frac{1}{U} \frac{\partial U}{\partial x'} + \frac{\tan \phi \tan \gamma'}{v} \frac{\partial v}{\partial x'}}{\csc \theta' + \tan \phi \sec \theta'} \quad (2-12)$$

b. Properties of the group ray curvature

The ray curvature of a wave packet defined by Equation (2-12) exhibits some very remarkable properties. Under

various conditions the trigonometric terms of the equation can become infinite or have indeterminate forms. The value of κ_G approaches zero as θ' approaches a direction either parallel or perpendicular to the wave speed contours, provided γ' does not have a direction parallel to the contours. This means that given a sufficiently long path, refraction tends to turn the wave packet so that it is directed either parallel or perpendicular to the wave speed contours. In the limit θ' or γ' together approach a direction parallel to the wave speed contours, κ_G approaches a finite nonzero number. As γ' , but not θ' , approaches a direction parallel to the wave speed contours, κ_G approaches infinity and the wave group undergoes total reflection.

For parallel wave speed contours, the value of θ' at a reflection point can be determined using Snell's law, which is the integrated form of the ray curvature expression. Snell's law for a wave packet can be written

$$\frac{\sin \theta'}{U \cos (\theta' - \gamma')} = C_s \quad (2-13)$$

where C_s is a constant. When the wavelet direction is parallel to the wave speed contours (Equation 2-13) becomes

$$\sin \theta' = \pm U C_s \sin \theta' \quad (2-14)$$

where the sign is positive or negative depending on whether $\sin \gamma' = \pm 1$. Equation (2-14) holds for all values of U and C_s only if θ' is an integral multiple of 180° . Accordingly, at a reflection point the wavelet direction becomes parallel to the wave speed contours, the wave packet direction becomes perpendicular to the contours, the geometric group velocity goes to zero, and the ray curvature becomes infinite.

c. Wavelet directions

Both the wave packet and wavelet directions must be computed in determining each point of a ray path. Equation (2-1) can be used to calculate γ . However, since the packet and wavelets travel with different velocities, the incremental distances by which they advance are different. The wavelet incremental distance must extend to the wave speed contour reached by the wave packet. This is illustrated in Figure (2-1) where the wave speed contours are assumed to be locally parallel. From the figure, it is seen that

$$d\lambda_v = \frac{\cos \theta'}{\cos \gamma'} d\lambda_G \quad (2-15)$$

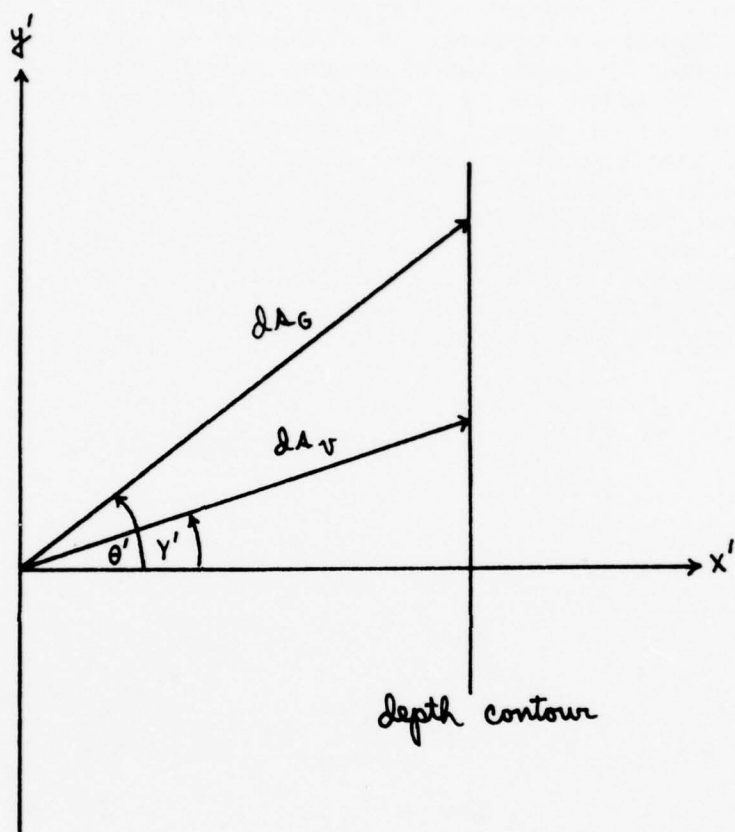


Figure (2-1). RELATIONSHIP BETWEEN THE WAVE PACKET AND WAVELET INCREMENTAL DISTANCES

Equation (2-15) is well behaved except at reflection points where $\cos \gamma' = 0$ and $\kappa_G = \infty$.

The wavelet direction can be calculated using Snell's law with phase velocity. This offers the advantage that the wavelet incremental distance, which must be determined in the ray curvature method, is replaced by the wave speed which is computed at each point of the packet trajectory.

In order to be consistent with the rest of the computations, it is necessary to present Snell's law in a form such that the incident angle is defined with respect to the positive x' -axis. To do this, a number of rules are employed where the subscripts n and $(n + 1)$ refer to consecutive points of a ray and n is a positive integer. The first step, if necessary, is to successively add or subtract 360° from the incident wavelet angle until it is within the range $0 \leq \gamma_n' < 360^\circ$. Then, when Snell's law is given by

$$\gamma_{n+1}^* = \arcsin \left(\frac{v_{n+1}}{v_n} \sin \gamma_n' \right) \quad (2-16)$$

where $-90^\circ \leq \gamma_{n+1}^* \leq 90^\circ$, the angle γ_{n+1}' is defined by the following scheme.

$$\gamma_{n+1}' = \begin{cases} \gamma_{n+1}^* , & \gamma_n' \leq 90^\circ \\ 180^\circ - \gamma_{n+1}^* , & 90^\circ < \gamma_n' \leq 270^\circ \\ 360^\circ + \gamma_{n+1}^* , & \gamma_n' > 270^\circ \end{cases} \quad (2-17)$$

2.2 Spatial Derivatives of G , U , v , and h . In this section relations are presented for connecting the partial derivatives of G , U , v , and h .

a. Determination of h and its partial derivatives

For each ray point the water depth h is interpolated from a quadratic surface equation which is fitted to the water depths at 12 grid points as illustrated in Figure (2-2). The use of a quadratic surface makes it possible to evaluate second derivatives which are required in calculating the wave height. The surface is approximated by the general quadratic equation (Dobson, 1967)

$$h = E_1 + E_2 x + E_3 y + E_4 x^2 + E_5 xy + E_6 y^2 \quad (2-18)$$

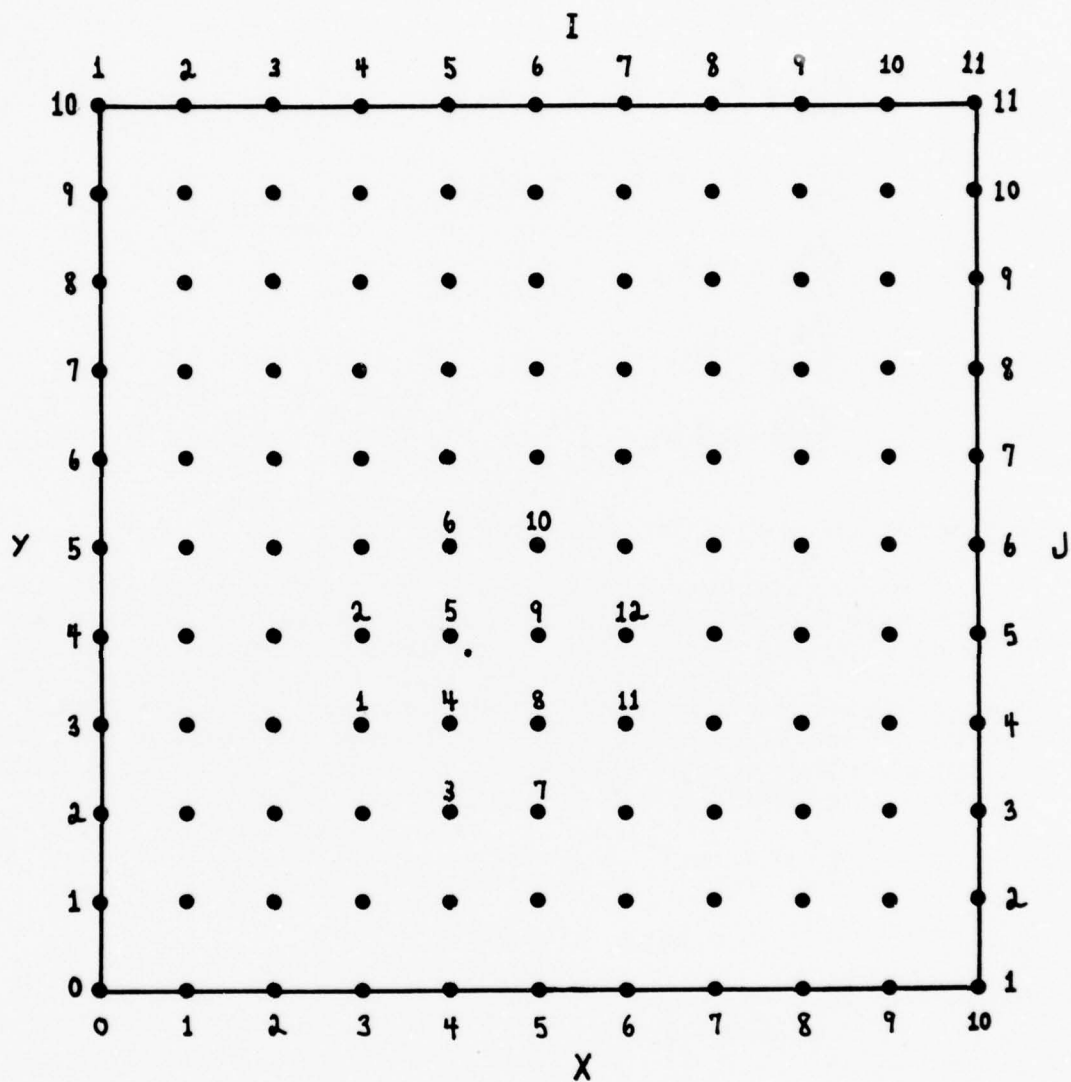


Figure (2-2). SELECTION OF WATER DEPTHS ABOUT
RAY POINT $(X,Y) = (4.2, 3.8)$

where the coefficients E are determined by fitting the equation by the method of least squares to the 12 water depth values. The partial derivatives of h are readily determined from Equation (2-18).

$$\frac{\partial h}{\partial x} = E_2 + 2E_4 x + E_5 y \quad (2-19)$$

$$\frac{\partial h}{\partial y} = E_3 + E_5 x + 2E_6 y \quad (2-20)$$

$$\frac{\partial^2 h}{\partial x^2} = 2E_4 \quad (2-21)$$

$$\frac{\partial^2 h}{\partial x \partial y} = E_5 \quad (2-22)$$

$$\frac{\partial^2 h}{\partial y^2} = 2E_6 \quad (2-23)$$

b. Rotation of axes to make computations

A reference xy -coordinate system aligned with respect to the water depth grid is used to tabulate the particulars of a wave packet trajectory. However, the calculations at each point of the trajectory are made in an $x'y'$ -coordinate system chosen such that the y' -derivatives of h are zero. The relationships between the coordinate systems, a ray, and a depth contour are illustrated in Figure (2-3). Equations relating the coordinate systems are given by

$$x' = x \cos \alpha + y \sin \alpha \quad (2-24)$$

$$y' = -x \sin \alpha + y \cos \alpha \quad (2-25)$$

$$\theta' = \theta - \alpha \quad (2-26)$$

$$Y' = Y - \alpha \quad (2-27)$$

The angle α by which the x' -axis is rotated with respect to the x -axis is given by

$$\tan \alpha = \frac{\partial h}{\partial y} / \frac{\partial h}{\partial x} \quad (2-28)$$

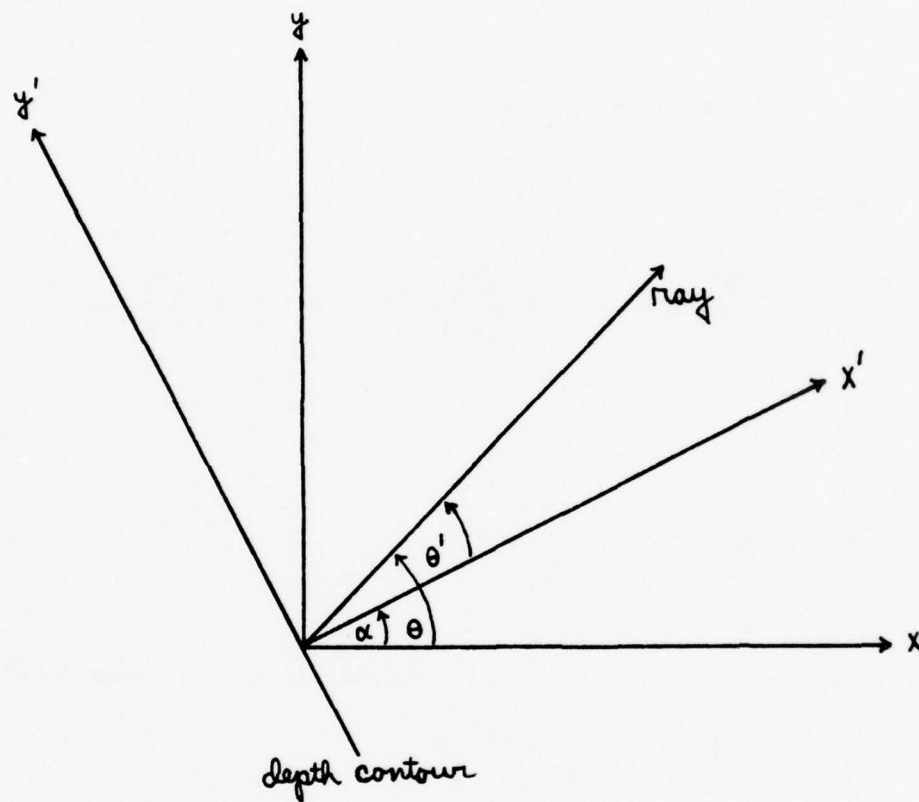


Figure (2-3). RELATIONSHIPS BETWEEN THE COORDINATE SYSTEMS, A RAY, AND A DEPTH CONTOUR

As a consequence, the positive x' -axis is perpendicular to the water depth contour in the direction of increasing h . For the assumption of locally parallel wave speed contours to be good, the variation in α between successive points of a ray must be small.

The partial derivatives of h in the $x'y'$ -coordinate system become

$$\frac{\partial h}{\partial x'} = \frac{\partial h}{\partial x} \cos \alpha + \frac{\partial h}{\partial y} \sin \alpha \quad (2-29)$$

$$\frac{\partial h}{\partial y'} = 0 \quad (2-30)$$

$$\frac{\partial^2 h}{(\partial x')^2} = \frac{\partial^2 h}{\partial x^2} \cos^2 \alpha + 2 \frac{\partial^2 h}{\partial x \partial y} \sin \alpha \cos \alpha + \frac{\partial^2 h}{\partial y^2} \sin^2 \alpha \quad (2-31)$$

$$\frac{\partial^2 h}{(\partial y')^2} = 0 \quad (2-32)$$

$$\frac{\partial^2 h}{\partial x' \partial y'} = 0 \quad (2-33)$$

Alternative expressions for Equations (2-29) and (2-31) are

$$\frac{\partial h}{\partial x'} = \left[\left(\frac{\partial h}{\partial x} \right)^2 + \left(\frac{\partial h}{\partial y} \right)^2 \right]^{\frac{1}{2}} \quad (2-34)$$

$$\frac{\partial^2 h}{(\partial x')^2} = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \quad (2-35)$$

c. Derivatives of v

As a convenience in the computations, the spatial derivatives of v are expressed in terms of the spatial derivatives of h . For linear theory (Lamb, 1932) the phase speed of a gravity water wave can be defined

$$v = \frac{1}{a} \tanh \frac{I}{2} \quad (2-36)$$

where

$$a = \frac{2\pi}{gT} \quad (2-37)$$

$$b = \frac{T}{4\pi} \quad (2-38)$$

$$I = \frac{h}{bv} \quad (2-39)$$

and g is the acceleration due to gravity. For a given wavelet period, a and b are constants. It can be shown (Wilson, 1966; Dobson, 1967; Breeding, 1972) that

$$\frac{\partial v}{\partial x} = W \frac{\partial h}{\partial x} \quad (2-40)$$

$$\frac{\partial v}{\partial y} = W \frac{\partial h}{\partial y} \quad (2-41)$$

where

$$W = \frac{v(1-a^2v^2)}{[2abv^2 + h(1-a^2v^2)]} \quad (2-42)$$

Further

$$\frac{\partial v}{\partial x'} = W \frac{\partial h}{\partial x'} \quad (2-43)$$

Differentiation of Equation (2-43) yields after some simplification

$$\frac{\partial^2 v}{(\partial x')^2} = W \left[\frac{\partial^2 h}{(\partial x')^2} + \gamma \left(\frac{\partial h}{\partial x'} \right)^2 \right] \quad (2-44)$$

where

$$\gamma = - \frac{4abv^2}{[2abv^2 + h(1-a^2v^2)]^2} \quad (2-45)$$

d. Derivatives of U

For linear wave theory the collinear group speed of a gravity water wave can be defined

$$U = \frac{1}{2} \left(1 + \frac{I}{\sinh I} \right) v \quad (2-46)$$

The spatial derivatives of U can be expressed

$$\frac{\partial U}{\partial x'} = \frac{\cosh I}{2} \left[(\sinh I + 1) \frac{\partial v}{\partial x'} + v \frac{\partial I}{\partial x'} (1 - I \coth I) \right] \quad (2-47)$$

$$\begin{aligned} \frac{\partial^2 U}{(\partial x')^2} = & \frac{\cosh I}{2} \left\{ - \left[(\sinh I + 1) \frac{\partial v}{\partial x'} + v \frac{\partial I}{\partial x'} (1 - I \coth I) \right] \coth I + \right. \\ & + (\sinh I + 1) \frac{\partial^2 v}{(\partial x')^2} + \frac{\partial v}{\partial x'} \frac{\partial I}{\partial x'} (2 + \cosh I - I \coth I) + \\ & \left. + v \left[\frac{\partial^2 I}{(\partial x')^2} (1 - I \coth I) + \left(\frac{\partial I}{\partial x'} \right)^2 \cosh I (I \cosh I - \cosh I) \right] \right\} \end{aligned} \quad (2-48)$$

where

$$\frac{\partial I}{\partial x'} = I \left(\frac{1}{k} \frac{\partial k}{\partial x'} - \frac{1}{v} \frac{\partial v}{\partial x'} \right) \quad (2-49)$$

$$\begin{aligned} \frac{\partial^2 I}{(\partial x')^2} = & \frac{1}{I} \left(\frac{\partial I}{\partial x'} \right)^2 + I \left\{ \frac{1}{k} \left[\frac{\partial^2 k}{(\partial x')^2} - \frac{1}{k} \left(\frac{\partial k}{\partial x'} \right)^2 \right] - \right. \\ & \left. - \frac{1}{v} \left[\frac{\partial^2 v}{(\partial x')^2} - \frac{1}{v} \left(\frac{\partial v}{\partial x'} \right)^2 \right] \right\} \end{aligned} \quad (2-50)$$

e. Derivatives of G

Equation (2-11) can be restated as

$$\frac{\partial G}{\partial x'} = \rho (\cos \phi \frac{\partial U}{\partial x'} + \sigma \frac{\partial v}{\partial x'}) \quad (2-51)$$

where

$$\rho = (1 + \tan \phi \tan \theta')^{-1} \quad (2-52)$$

$$\sigma = \frac{U}{v} \sin \phi \tan \gamma' \quad (2-53)$$

The second spatial derivative of G is given by

$$\begin{aligned} \frac{\partial^2 G}{(\partial x')^2} = & \rho \left(\cos \phi \frac{\partial^2 U}{(\partial x')^2} + \sigma \frac{\partial^2 v}{(\partial x')^2} \right) + \left(\cos \phi \frac{\partial \rho}{\partial x'} - \right. \\ & \left. - \rho \sin \phi \frac{\partial \phi'}{\partial x'} \right) \frac{\partial U}{\partial x'} + \left(\rho \frac{\partial \sigma}{\partial x'} + \sigma \frac{\partial \rho}{\partial x'} \right) \frac{\partial v}{\partial x'} \end{aligned} \quad (2-54)$$

where

$$\frac{\partial \rho}{\partial x'} = -\rho^2 \left(\tan \theta' \sec^2 \phi \frac{\partial \phi'}{\partial x'} + \tan \phi \sec^2 \theta' \frac{\partial \theta'}{\partial x'} \right) \quad (2-55)$$

$$\frac{\partial \sigma}{\partial x'} = \sigma \left(\frac{1}{U} \frac{\partial U}{\partial x'} - \frac{1}{v} \frac{\partial v}{\partial x'} \right) + \frac{U}{v} \left(\cos \phi \tan \gamma' \frac{\partial \phi'}{\partial x'} + \sin \phi \sec^2 \gamma' \frac{\partial \gamma'}{\partial x'} \right) \quad (2-56)$$

The derivatives of γ' and θ' are defined by Equations (2-9) and (2-10), and

$$\frac{\partial \phi'}{\partial x'} = \frac{\partial \theta'}{\partial x'} - \frac{\partial \gamma'}{\partial x'} \quad (2-57)$$

2.3 Computation of Gravity Water Wave Packet Trajectories. In this section a method is presented for computing the surface trajectories of gravity water wave packets.

a. Determining the path

Successive points of a wave packet trajectory are found by iteration using Equation (2-12) for the ray curvature. The ray curvature is calculated on the assumption that the water depth contours are locally parallel in the vicinity of each point of the trajectory. At each point of the wave packet trajectory the wavelet direction is found using Snell's law with phase velocity as defined by Equations (2-16) and (2-17).

The coordinates of each ray point are defined by

$$x_{m+1} = x_m + D_m \cos \bar{\theta} \quad (2-58)$$

$$y_{m+1} = y_m + D_m \sin \bar{\theta} \quad (2-59)$$

$$\bar{\theta} = \frac{1}{2} (\theta_m + \theta_{m+1}) \quad (2-60)$$

$$\theta_{m+1} = \theta_m + \Delta\theta \quad (2-61)$$

$$\Delta\theta = \frac{1}{2} \left[(K_G)_m + (K_G)_{m+1} \right] D_m \quad (2-62)$$

$$D_m = G(\Delta t) / \text{GRID} \quad (2-63)$$

where D_n is the incremental distance in grid units between the points n and $(n + 1)$ of a ray, GRID is the grid unit distance in units consistent with G , and (Δt) is the time step.

b. Reflection points

Reflection points require special consideration. The waves are assumed to reflect if any of three criteria are satisfied. The first test for reflection is based on Snell's law with phase velocity. Reflection occurs if

$$\left| \frac{v_{m+1}}{v_m} \sin \gamma'_m \right| > 1 \quad (2-64)$$

As a reflection point is approached the ray curvature changes so quickly that calculations of the ray curvature by iteration may cease to converge. If convergence fails reflection is assumed if the following conditions are met.

$$\frac{v_{m+1}}{v_m} > 1 \quad (2-65)$$

$$|\tan \gamma''| > \tan 80^\circ$$

The first condition requires that the phase speed increases between the last two ray points, and the second condition requires that the wavelet direction be consistent with total reflection.

A third criterion is used to specify reflection points in order to maintain accuracy in calculating the wave

packet trajectory. Very near a reflection point, due to the rapidly changing ray curvature, the estimates of ray points can become erratic. Therefore, reflection is assumed if the following conditions are met

$$\begin{aligned} \frac{v_{m+1}}{v_m} &> 1 \\ |\tan \gamma'| &> \tan 89.5^\circ \\ |\tan \theta'| &< \tan 75^\circ \end{aligned} \quad (2-66)$$

When a reflection point is determined on the basis of either the second or third criteria, it is advisable to examine the printout to determine if the ray particulars are consistent with total reflection. The values of κ_G , γ' , θ' , and G should exhibit the behavior described in Section (2.1), b.

When a reflection occurs there is an option to either halt the wave packet trajectory or to continue it as a reflection. When the wave packet reflection option is chosen, the reflection angles are determined by the relations

$$\theta'_r = -\theta'_i + 180^\circ \quad (2-67)$$

$$\gamma'_r = -\gamma'_i + 180^\circ \quad (2-68)$$

The subscript r denotes the reflection angles, and the subscript i signifies the angles at the last ray point reached prior to where the reflection criterion is satisfied.

The ray curvature calculations are more likely to converge and the accuracy of the path is increased if there is a restriction on how much the ray direction can change between successive ray points. Accordingly, if γ' is within 15° of a direction for which the ray curvature is infinite, the time step between ray points is successively halved, as necessary, until $|\Delta\theta|$ is less than 1° . In the event it is necessary to reduce the time step to less than 0.5 seconds the ray is stopped.

2.4 Wave Height Calculations. Modification of the wave height due to refraction, shoaling, and energy dissipation is considered. The wave height H increases when adjacent rays converge and it decreases when the rays diverge; this effect is accounted for by the refraction coefficient K_R . The shoaling coefficient K_S accounts for the change in H due to the variation in the propagation speed of the wave packet. The loss in energy due to wave motion at the sea bottom is determined by the friction coefficient K_F .

a. Without energy dissipation

The average rate of energy transmission F can be defined

$$F = E \ell G \quad (2-69)$$

where E is the energy per unit area and ℓ is the perpendicular distance between rays. The energy is assumed to be conserved along the ray. Therefore

$$F_{j+1} = F_j \quad (2-70)$$

where j and $(j + 1)$ denote consecutive ray points. It is further assumed that E is proportional to H^2 . Accordingly, it follows from Equations (2-69) and (2-70) that

$$H_{j+1} = (K_S)_{j+1} (K_R)_{j+1} H_j \quad (2-71)$$

where $(K_S)_{j+1}$ and $(K_R)_{j+1}$ are the shoaling and refraction coefficients, respectively, between points j and $(j+1)$.

$$(K_S)_{j+1} = \left(\frac{G_j}{G_{j+1}} \right)^{\frac{1}{2}} \quad (2-72)$$

$$(K_R)_{j+1} = \left(\frac{\ell_j}{\ell_{j+1}} \right)^{\frac{1}{2}} \quad (2-73)$$

If H_0 is the initial wave height, then the wave height at the n -th point is

$$H_m = K_S K_R H_0 \quad (2-74)$$

where

$$K_S = (K_S)_1 (K_S)_2 \cdots (K_S)_m = \left(\frac{G_0}{G_m} \right)^{\frac{1}{2}} \quad (2-75)$$

$$K_R = (K_R)_1 (K_R)_2 \cdots (K_R)_m = \left(\frac{l_0}{l_m} \right)^{\frac{1}{2}} \quad (2-76)$$

b. With energy dissipation

To account for energy losses, Equation (2-70) can be restated

$$F_{j+1} = (K_F)_{j+1}^2 F_j \quad (2-77)$$

where $(K_F)_{j+1}$ is the friction coefficient between the points j and $(j+1)$. As a result, the relationship between the wave heights at consecutive ray points can be expressed by

$$H_{j+1} = (K_S)_{j+1} (K_R)_{j+1} (K_F)_{j+1} \quad (2-78)$$

In terms of the initial wave height, the wave height at the n -th point is given by

$$H_m = K_S K_R K_F H_0 \quad (2-79)$$

where

$$K_F = (K_F)_1 (K_F)_2 \cdots (K_F)_m = (K_F)_m (K_F)_m \quad (2-80)$$

2.5 Refraction Coefficient. In computing K_R it is convenient to define

$$\beta = \frac{l_m}{l_0} \quad (2-81)$$

where β is called the ray separation factor. Equation (2-76) for the refraction coefficient becomes

$$K_R = |\beta|^{-\frac{1}{2}} \quad (2-82)$$

a. Ray separation equation

In considering the refraction of monochromatic waves, Munk and Arthur (1952) have shown that β can be determined from a second-order differential equation called the ray separation equation. The equation can be stated

$$\frac{d^2\beta}{dt^2} + p \frac{d\beta}{dt} + q\beta = 0 \quad (2-83)$$

where t is time. For a wave packet trajectory

$$p = -2(\cos \theta \frac{\partial G}{\partial x} + \sin \theta \frac{\partial G}{\partial y}) \quad (2-84)$$

$$q = G(\sin^2 \theta \frac{\partial^2 G}{\partial x^2} - 2 \sin \theta \cos \theta \frac{\partial^2 G}{\partial x \partial y} + \cos^2 \theta \frac{\partial^2 G}{\partial y^2}) \quad (2-85)$$

The calculations for p and q can be simplified in the same manner as in calculating the wave packet ray curvature by using the $x'y'$ -coordinate system. Then, p and q reduce to

$$p = -2 \cos \theta' \frac{\partial G}{\partial x'} \quad (2-86)$$

$$q = G \sin^2 \theta' \frac{\partial^2 G}{(\partial x')^2} \quad (2-87)$$

b. Solution for parallel water depth contours

There is a simple solution to Equations (2-83), (2-86), and (2-87) when the water depth contours are everywhere parallel. Then, with the $x'y'$ -coordinate system defined as in Section 2.2, b, it can be shown that

$$\beta = \frac{\cos \theta_n'}{\cos \theta_0'} \quad (2-88)$$

where the subscript 0 denotes the initial value and the subscript n depicts the value at the n -th ray point. The time derivative of Equation (2-88) is

$$\frac{\partial \beta}{\partial t} = - \frac{\sin^2 \theta_m}{\cos \theta_0} \frac{\partial G}{\partial x'} \quad (2-89)$$

where Equations (2-8) and (2-10) are used.

c. Numerical solutions of the ray separation equation

Several numerical methods can be used to solve the ray separation equation. The assumption is made that the water depth contours are locally parallel in the neighborhood of each ray point; therefore, p and q are defined by Equations (2-86) and (2-87). An easy to use fourth order finite difference solution to Equation (2-83) is the Fox method (Salvadori and Baron, 1961). However, this method has the disadvantage that the time step must be constant between successive ray points. When p and q do not change much between ray points, the general solution of a homogeneous second-order differential equation with constant coefficients (Wylie, 1951) can be used to solve the ray separation equation. This solution has 3 cases depending on the value of $(p^2 - 4q)$. The value of β at each new ray point is found using the values of p and q at the last point. There is usually little difference between the results obtained by this method and the Fox method.

A numerical method which does not require a constant time step and which better accounts for the variation of p and q along a ray is the Runge-Kutta method. This method is stable. It is also self-starting, i.e., values at the previous point are used to find values at the next point (Romanelli, 1960). For these reasons, the Runge-Kutta method was selected for the solution of the ray separation equation.

In order to use the Runge-Kutta method, Equation (2-83) is reduced to a system of first-order equations.

$$\frac{\partial \beta}{\partial t} = u \quad (2-90)$$

$$\frac{\partial u}{\partial t} = -(pu + q\beta) \quad (2-91)$$

Both fourth and fifth order solutions of β are obtained. The initial conditions are the values of β and $d\beta/dt$ at the first ray point. The latter is estimated using Equation (2-89). The solutions require the values of (p_n, q_n) at the n -th ray point and the values (p_{n+1}, q_{n+1}) at the $(n+1)$ -th ray point. Further, the values of (p_1, q_1) , (p_2, q_2) , (p_3, q_3) , (p_4, q_4) , and (p_5, q_5) are needed along the ray at points intermediate to the ray points. They are determined, respectively, at time intervals of $(\Delta t)/3$, $(\Delta t)/4$, $0.45573725(\Delta t)$, $2(\Delta t)/3$, and

0.8(Δt) beyond the n-th ray point where Δt is the time step in the calculations.

A fourth order Runge-Kutta method with a minimum truncation error bound is given by Ralston (1962). The solution for the ray separation equation becomes

$$\beta_{m+1} = \beta_m + 0.17476028 K_1 - 0.55148066 K_2 + 1.20553560 K_3 + 0.17118478 K_4 \quad (2-92)$$

$$\left(\frac{\partial \beta}{\partial t}\right)_{m+1} = \left(\frac{\partial \beta}{\partial t}\right)_m + 0.17476028 L_1 - 0.55148066 L_2 + 1.20553560 L_3 + 0.17118478 L_4 \quad (2-93)$$

where

$$K_1 = (\Delta t) \left(\frac{\partial \beta}{\partial t}\right)_m \quad (2-94)$$

$$L_1 = -(\Delta t) \left[p_m \left(\frac{\partial \beta}{\partial t}\right)_m + q_m \beta_m \right] \quad (2-95)$$

$$K_2 = (\Delta t) \left[\left(\frac{\partial \beta}{\partial t}\right)_m + 0.4 L_1 \right] \quad (2-96)$$

$$L_2 = -(\Delta t) \left[p_2 \left(\left(\frac{\partial \beta}{\partial t}\right)_m + 0.4 L_1 \right) + q_2 (\beta_m + 0.4 K_1) \right] \quad (2-97)$$

$$K_3 = (\Delta t) \left[\left(\frac{\partial \beta}{\partial t}\right)_m + 0.29697761 L_1 + 0.15875964 L_2 \right] \quad (2-98)$$

$$L_3 = -(\Delta t) \left[p_3 \left(\left(\frac{\partial \beta}{\partial t}\right)_m + 0.29697761 L_1 + 0.15875964 L_2 \right) + q_3 (\beta_m + 0.29697761 K_1 + 0.15875964 K_2) \right] \quad (2-99)$$

$$K_4 = (\Delta t) \left[\left(\frac{\partial \beta}{\partial t}\right)_m + 0.21810040 L_1 - 3.05096516 L_2 + 3.83286476 L_3 \right] \quad (2-100)$$

$$L_4 = -(\Delta t) \left[p_{m+1} \left(\left(\frac{\partial \beta}{\partial t}\right)_m + 0.21810040 L_1 - 3.05096516 L_2 + 3.83286476 L_3 \right) + q_{m+1} (\beta_m + 0.21810040 K_1 - 3.05096516 K_2 + 3.83286476 K_3) \right] \quad (2-101)$$

A disadvantage of the Runge-Kutta method is that there is no simple means for estimating the truncation error (Milne, 1953). One procedure for controlling the error is to compute both fourth and fifth order solutions of β and to adjust the time step so that the two estimates differ by less than an arbitrary amount.

A fifth order Runge-Kutta method is given by Milne (1953). The fifth order solutions for β and $d\beta/dt$ are

$$\beta_{m+1}^{(5)} = \beta_m + \frac{1}{192} (23 K_1 + 125 K_6 - 81 K_8 + 125 K_9) \quad (2-102)$$

$$\left(\frac{d\beta}{dt}\right)_{m+1}^{(5)} = \left(\frac{d\beta}{dt}\right)_m + \frac{1}{192} (23 L_1 + 125 L_6 - 81 L_8 + 125 L_9) \quad (2-103)$$

where

$$K_5 = (\Delta t) \left[\left(\frac{d\beta}{dt}\right)_m + \frac{L_1}{3} \right] \quad (2-104)$$

$$L_5 = -(\Delta t) \left[p_1 \left(\left(\frac{d\beta}{dt}\right)_m + \frac{L_1}{3} \right) + q_1 \left(\beta_m + \frac{K_1}{3} \right) \right] \quad (2-105)$$

$$K_6 = (\Delta t) \left[\left(\frac{d\beta}{dt}\right)_m + \frac{6L_5 + 4L_1}{25} \right] \quad (2-106)$$

$$L_6 = -(\Delta t) \left[p_2 \left(\left(\frac{d\beta}{dt}\right)_m + \frac{6L_5 + 4L_1}{25} \right) + q_2 \left(\beta_m + \frac{6K_5 + 4K_1}{25} \right) \right] \quad (2-107)$$

$$K_7 = (\Delta t) \left[\left(\frac{d\beta}{dt}\right)_m + \frac{15L_6 - 12L_5 + L_1}{4} \right] \quad (2-108)$$

$$L_7 = -(\Delta t) \left[p_{m+1} \left(\left(\frac{d\beta}{dt}\right)_m + \frac{15L_6 - 12L_5 + L_1}{4} \right) + q_{m+1} \left(\beta_m + \frac{15K_6 - 12K_5 + K_1}{4} \right) \right] \quad (2-109)$$

$$K_8 = (\Delta t) \left[\left(\frac{d\beta}{dt}\right)_m + \frac{8L_7 - 50L_6 + 90L_5 + 6L_1}{81} \right] \quad (2-110)$$

$$L_8 = -(\Delta t) \left[p_4 \left(\left(\frac{d\beta}{dt}\right)_m + \frac{8L_7 - 50L_6 + 90L_5 + 6L_1}{81} \right) + q_4 \left(\beta_m + \frac{8K_7 - 50K_6 + 90K_5 + 6K_1}{81} \right) \right] \quad (2-111)$$

$$K_9 = (\Delta t) \left[\left(\frac{d\beta}{dt}\right)_m + \frac{8L_7 + 10L_6 + 36L_5 + 6L_1}{75} \right] \quad (2-112)$$

$$L_9 = -(\Delta t) \left[p_5 \left(\left(\frac{\partial \beta}{\partial t} \right)_m + \frac{8L_7 + 10L_6 + 36L_5 + 6L_1}{75} \right) + q_5 \left(\beta_m + \frac{8K_7 + 10K_6 + 36K_5 + 6K_1}{75} \right) \right] \quad (2-113)$$

The difference between the fourth and fifth order solutions of β and $d\beta/dt$ are

$$\epsilon_\beta = \beta_{m+1} - \beta_{m+1}^{(5)} \quad (2-114)$$

$$\epsilon_{\beta t} = \left(\frac{\partial \beta}{\partial t} \right)_{m+1} - \left(\frac{\partial \beta}{\partial t} \right)_{m+1}^{(5)} \quad (2-115)$$

In the calculations both $|\epsilon_\beta|$ and $|\epsilon_{\beta t}|$ are monitored. If either is greater than or equal to an arbitrary constant (determined as an input parameter) the time step is halved, the corresponding $(n+1)$ -th ray point is found, and the β and $d\beta/dt$ calculations are repeated. This process continues, as necessary, until both $|\epsilon_\beta|$ and $|\epsilon_{\beta t}|$ are less than the arbitrary constant. If the time step is reduced to less than 0.5 seconds the ray is stopped.

d. Reflection points

The numerical solutions of the ray separation equation have not produced satisfactory results near reflection points. This is possibly due to the rapid change in p which approaches infinity as the reflection point is approached. The problem occurs when the wavelet direction is within 15° of being parallel to the wave speed contours. Accordingly, for this narrow region, Equations (2-88) and (2-89) for parallel water depth contours are used to evaluate β and $d\beta/dt$. The equation for β is not only well behaved but approaches a constant value at the reflection point.

e. Caustics and focal points

The value of β is monitored along a ray. If the value becomes zero or negative a focal point or caustic is located. In this case the ray is stopped.

2.6 Friction Coefficient. Energy dissipation of the waves due to bottom friction is considered. The friction coefficient is determined using a method based on the theory of Putnam and Johnson (1949) and Bretschneider and Reid (1954). Other energy dissipation methods can be substituted if desired.

In this work the friction factor c_f is defined following Jonsson (1966)

$$\tau = \frac{1}{2} c_f \rho_f u_m^2 \quad (2-116)$$

where τ is the tangential stress per unit area at the bottom, ρ_f is the density of the fluid, and u_m is the maximum velocity of the fluid at the bottom. The definition for τ given by Putnam and Johnson (1949) does not contain the factor $\frac{1}{2}$. When Equation (2-116) is used the friction coefficient becomes

$$K_F = \frac{(K_F)_m}{F(K_F)_m (\Delta A_G)_{n+1} + 1} \quad (2-117)$$

where $(K_F)_m$ is defined by Equation (2-80), $(\Delta s_G)_{n+1}$ is the incremental distance between the ray points n and $(n+1)$, and

$$F = \left(\frac{8\pi^2}{3g} \right) \left(\frac{c_f H_0}{U_0} \right) \left(\frac{K_S}{T \sinh kh} \right)^3 \quad (2-118)$$

2.7 Wave Breaking Criterion. In the program there is an option to determine if the waves break. When this option is chosen, the waves are assumed to break when the following relation is satisfied.

$$\frac{H}{\lambda} > \frac{1}{7} \tanh kh \quad (2-119)$$

CHAPTER III THE COMPUTER PROGRAM

3.1 Description of the Computer Program. The computer program is written in Fortran IV for the Control Data Cyber 70 computer systems and plotters which are compatible with the Calcomp plotting systems. With the exception of the Calcomp subroutines, a description of each program subroutine is presented. The reader is referred to a Calcomp reference manual for descriptions of the Plot, Symbol, and Number subroutines (California Computer Products, Inc., 2411 West La Palma, Anaheim, CA. 92801). With the exception of the plotting subroutines, each one is written so that it is possible to follow the subroutine listing. It will be helpful to make use of the definitions of the symbols presented with the Notation at the end of this work.

Including the 100 by 100 array for the water depth grid the program requires 30976 words (base 10). In order to reduce the size of the computer program card deck and to shorten the program listing, several program statements have frequently been combined on one computer card. This is done by separating the statements by a blank, a dollar sign, and another blank.

There is often a need to prepare forecasts for a number of different water depth grids. Further, it is frequently desirable to make a number of forecasts for the same water depth grid. To make it easier to handle the input data for these situations, the water depth grids (XYGRID) are all stored on one input file. Each grid has its own name, e.g., XYGOM1. The rest of the input parameters (RAYDAT) are stored on another input file, and the data for each run has a name, e.g., RAYDAT1. When using the program an appropriate set of control cards is used to access the input data. If this feature is not desired, the READ statements in MAIN should be changed so that the same input file is used for all the input data.

When checks of the program are made or if there are modifications to the program, it should be noted that English units are used internally in the program for the calculations. In addition, the input and output wave packet and wavelet directions are defined as the directions from which the waves come with respect to true north. Before making calculations these angles are transformed using the following relationships

$$\theta_C = \text{CNVRS}A - \theta_N + 180 \quad (3-1)$$

$$\gamma_C = \text{CNVRS}A - \gamma_N + 180 \quad (3-2)$$

where the subscript C refers to the calculation coordinate system, the subscript N denotes the true north coordinate

system, and CNVRS is the direction of the positive x-axis of the water depth grid with respect to true north. The angles are in degrees.

3.2 PROGRAM MAIN. The MAIN PROGRAM controls the input, plots, and calculations for all the rays. To begin with, the values of MMAX, LI, and CORI are assigned values, and LII is calculated. Two statements are used to read numbers used in the surface fitting procedure in SURFCE. Then, descriptive information is read which is used for all plots. Next, two read statements are used to obtain input parameters for a specific plot. If MOE \neq 0 there is a conversion from Metric to English units. The value of CIN is changed from seconds to hours, and the values of AMM, ANN, DY, and SCLI are determined. Next TITLE is called.

If NXCMAT = 0 the water depths are read and stored in CMAT. If NXCMAT \neq 0 no water depths are read, and the depth grid used in the previous plot is used again. If NCO \neq 0 sounding depths for the plot are read and NUMCON is called. If NCO = 0 no sounding depths are read; in this case there must be no sounding depth card. SHORE is called if NSH \neq 0.

Next, the input parameters for a given ray are read. If MOE \neq 0 there is a conversion from Metric to English units. The values of SDLTAT and WL are defined, and the computational values of the wave packet and wavelet directions are determined. MAXQ is initialized to one and FUD, BRK, REFLCT, RFLBUM, REFRCT, RFRBUM, FLAGR, FLAG3, IFLG, and α are initialized to zero. If COL \neq 0 the plotter will pause before a ray is plotted. The values of A and AV are changed from degrees to radians. Then RAYN is called. After all rays for a given plot are determined the comment: THIS IS THE END is written on the output.

3.3 Subroutine TITLE. TITLE is called by MAIN to draw labels and straight line borders on a plot. The labels consist of PROJCT, DATE1, DATE2, SCL, CIN, NPLOT, and DIR. If NAX \neq 0, AXIS2 is called.

3.4 Subroutine AXIS2. AXIS2 is called by TITLE to prepare xy-axes for the plot. The axes are calibrated with tick marks, and the origin and every fifth tick mark are numbered. Finally, each axis is labeled.

3.5 Subroutine NUMCON. If NCO > 0, NUMCON is called by MAIN to locate NCO sounding water depths on a plot. The sounding depths are stored in the array CONTUR. If MOE \neq 0 the sounding depths are converted from Metric to English values before the calculations are made. The search for the sounding depths begins one grid unit from the end of a column starting with the second y-column. The column is searched separately for each depth. If necessary, the sounding depths are located by linear interpolation.

After all the sounding depths for a given column are found, they are drawn at their respective locations on the plot. In the event $MOE \neq 0$ the English values are converted back to Metric values before being drawn on the plot.

This process is repeated for additional y-columns where the next column is determined by adding NNSKIP to the number of the previous y-column. The process stops when the y-column selected is greater than (NN-1). A restriction on the use of this subroutine is that $(\partial h / \partial x)|_{h=0} \geq 0$ for the entire depth grid.

3.6 Subroutine SHORE. If NSH $\neq 0$, SHORE is called by MAIN to draw the shoreline on the plot. Beginning with the first y-column in CMAT, each column is searched for the location of the zero water depth. The search in each column begins with the maximum value of x, and if necessary, the point of zero water depth is found by linear interpolation. The shoreline is the line drawn connecting these points. To use this subroutine it is necessary that $(\partial h / \partial x)|_{h=0} \geq 0$.

3.7 Subroutine RAYN. This subroutine is called by MAIN to control the calculation of the wave packet trajectory and the wave particulars along the trajectory. Initially, NDP, NFK, NGO, and FLAG1 are set equal to one. Also, KREST, KCIN, and RCOUNT are put equal to zero. SURFCE is called to calculate ray particulars for the first ray point. FLAG1 and INUM are set equal to zero. The saved values of α , γ , h, v, u, and G are initialized. MOVE is called to calculate the initial value of D, and HEIGHT is called to determine the initial wave height. The travel time is initialized to zero, and the initial wave packet and wavelet directions are converted to degrees and to values measured with respect to true north for later printout. Then the value of NPT is checked to determine how much printout is desired.

If NPT = 0 printout occurs at the first and last ray points. When NPT $\neq 0$, PCD is called to calculate PCTDIF and printout occurs for selected ray points depending upon the value of SK. The ray parameters which appear in the output depend upon the value of NPT. The procedure employed to obtain the output when NPT $\neq 0$ is presented below. However, there is little difference in the routine used to obtain output when NPT = 0.

Printout occurs for the first ray point or if the number of the ray point is an integral multiple of SK. Then α is changed to degrees, and if MOE $\neq 0$ the English values of the ray parameters are changed to Metric values. If the ratio of FUD to LI has no remainder, page and column headings are written. First, the page heading is written. Next, if the ray is at the first point the initial value of $d\beta/dt$ is written. In addition, if MOE = 0 the printout contains: THE OUTPUT IS IN ENGLISH UNITS. H, HGT(FEET). G, U, V(FEET/SECOND). If instead MOE $\neq 0$ the printout contains: THE

OUTPUT IS IN METRIC UNITS. H, HGT(METER). G, U, V(METER/SECOND). Then the column headings are written.

FUD is increased by one and the ray particulars are written. One of three formats is used. If RFLBUM \neq 0 (determined in HEIGHT) the format for a reflection breakup of the time step interval is chosen. Further, RFLBUM is set equal to zero. If RFLBUM = 0 and RFRBUM \neq 0 (determined in HEIGHT) the format for a refraction breakup is used and RFRBUM is set equal to zero. The remaining format is used if there has not been a breakup of the time step interval.

The value of α is changed back to radians. If MOE \neq 0 the ray particulars which were changed to Metric values are converted back to English values for use in the calculations. The values of the ray point number, the coordinates of the ray point, and the number of breakup intervals are saved. Then STORE is called.

After returning from STORE, if MIT = 2 (determined in MOVE), if NPT \neq 0, and if the ratio of FUD to LI has no remainder, then page and column headings are written. FUD is increased by one, and the printed output contains the statement: PACKET CURVATURE AVERAGED. If after returning from STORE, NPT = 0 or MIT = 1 the previous write statements are omitted.

When at the first ray point the value of NDP is checked. If NDP = 2 (determined in SURFCE) the printed output is: RAY REACHED SHORE. The ray is stopped and the program returns to MAIN. If NDP = 1 the number of the ray point is increased by one, and the dimension of the AX and AY arrays is checked (described below). For points beyond the first the value of NGO is checked. If NGO = 1 the ray point number is increased by one, and the size of the AX and AY arrays is checked. If NGO \neq 1 the ray is stopped, the printed output contains: RAY REACHED GRID BOUNDARY, and the program returns to MAIN.

Before the next ray point is calculated a check is made to determine if there is additional storage space in the AX and AY arrays. If the sum of the number of ray points and the number of tick marks (if any) exceeds MMAX the ray is stopped. The statement in the printed output is: DIMENSION OF OUTPUT-ARRAYS EXCEEDED. The ray particulars for the last point are written if they have not been previously written. Page and column headings appear when appropriate, and the format used is determined as explained above for the output of other ray points. The program returns to MAIN.

If the dimension of the output arrays is not exceeded, the value of G is saved and MOVE is called to find the next ray point. After the return to RAYN the ray is stopped if NDP \neq 1 or MIT = 3, 4, 5, 6, 7, or 8. The ray particulars for the last ray point are written unless they have already appeared in the output. In addition, one of the following descriptive printouts occurs. If NDP \neq 1 (determined in SURFCE) the printout is: RAY REACHED SHORE. When MIT = 3 (determined in MOVE) the printout is: PACKET CURVATURE

ITERATION NOT CONVERGING. If MIT = 4 (determined in HEIGHT) the statement is: CAUSTIC OR FOCAL POINT. When MIT = 5 (determined in HEIGHT) the printout contains: WAVE BREAKS. When MIT = 6 (determined in MOVE), which can occur when ROP = 0, there is no descriptive printout. If MIT = 7 (determined in MOVE) the printout is: REFLECTION HANG-UP. Finally, if MIT = 8 (determined in HEIGHT) the printout is: BREAKUP TIME STEP LESS THAN 0.5 SECOND.

After the return from MOVE to RAYN, the ray continues if NDP = 1 and MIT = 1 or 2 (determined in MOVE). The travel time is computed. The values of α , κ_G , h , D , G , v , U , H , K_S , K_F , and K_R are saved in case printout is required at the last ray point. The wave packet and wavelet angles are converted to degrees and are defined with respect to true north. Then, as described above, PCD is called if desired, the ray particulars are written if required, and MOVE is called if appropriate to find the next ray point.

3.8 Subroutine MOVE. MOVE is called by RAYN to determine the path of the wave packet. NUMT and MIT are initialized to one. The values of the geometric group speed and ray curvature are saved for use if the time step interval is divided into smaller intervals. At the second ray point the value of κ_G is saved; at other points the values of the ray curvature and the average ray curvature are saved.

The value of the incremental distance to the next ray point is computed. If at the first ray point the program returns to MOVE. At the second ray point the average ray curvature is set equal to the ray curvature obtained at the first point. For points beyond the second this latter step is ignored.

A check is made to determine if there is a breakup of the time step interval due to the calculation of the ray separation factor. If so, REFRCT was set equal to one in HEIGHT. Further, a check is made to determine if there is or should be a breakup of the time step interval due to a reflection point. If REFLCT \neq 0 a reflection breakup has occurred; if $|\tan \gamma'| > \tan 75^\circ$ and $|\tan \theta'| < \tan 75^\circ$ a reflection breakup should occur. Accordingly, if REFRCT \neq 0 or REFLCT \neq 0 or $|\tan \gamma'| \leq \tan 75^\circ$ or $|\tan \theta'| \geq \tan 75^\circ$ the iteration for the next point begins. Otherwise, REFLCT and RFLBUM are set equal to one before the iteration for the next point begins. The value of RFLBUM determines the format for the printed output of the ray particulars.

A maximum of fifty iterations can occur in locating a new point. On the first iteration the average ray direction to and the position of the next point are estimated using the ray curvature of the present point. Beyond the first iteration the average of the ray curvatures at the present point and the approximated next point is used to obtain a new estimate of the ray point. With each iteration SURFCE is called to calculate the ray curvature and other ray particulars at the

estimated position of the next point. After the return to MOVE, the wavelet direction γ' is computed.

If $\text{FLAG2} \neq 0$ (determined in SURFCE) a reflection occurs due to Snell's law with phase velocity. In this case, $\text{REF} = 1$ and there is a reflection (described below). If $\text{FLAG2} = 0$, DUD is calculated. If $\text{MIT} > 2$ the program returns to RAYN. If $\text{MIT} = 2$ (described below) a check is made to determine if the ray is too close to a reflection point (described below). If $\text{MIT} = 1$ the value of NDP is examined. If $\text{NDP} = 2$ (determined in SURFCE), $h \leq 0$ so the program returns to RAYN. If $\text{NDP} \neq 2$ the ray curvature at the latest estimate of the new point is averaged with the value at the present point; the average is used in the next approximation of the new point. The average ray curvature is saved on the 48th and 49th iterations.

The check for the convergence of the ray curvature depends upon both the number of the ray point and the number of the iteration. For the first or second ray point and on the first iteration the average ray curvature is saved and the second iteration begins. Beyond the first iteration successive curvature averages are checked, and if they differ by less than $0.00009/D$ convergence has occurred. The new point is checked to see if it is too close to a reflection point. If convergence has not occurred the iteration continues.

Beyond the second ray point and on the first iteration the ray curvature average for the estimated new point is compared with the ray curvature average of the present point. If the values differ by less than $0.00009/D$ convergence has occurred. On successive iterations curvature averages are checked to see if they differ by less than $0.00009/D$. When convergence occurs the check is made to determine if the new point is too close to a reflection point.

If convergence has not occurred after fifty iterations the ray curvatures on the 48th and 50th iterations are compared to see if they differ by less than $0.00009/D$. If so, the ray curvature is assumed to have converged to two values. This would happen if estimates of the new ray point alternate between two grid cells. For this situation $\text{MIT} = 2$, and the average of the ray curvature averages for the 49th and 50th iterations is determined and used to locate the next ray point. The point is checked to see if it is too close to a reflection point.

If after fifty iterations convergence is not achieved a check is made to find if the convergence failed because of a reflection point. This is possible since the ray curvature becomes infinite at a reflection point. Reflection is assumed if $\text{DUD} > 1$, i.e., the phase speed increases between the last two ray points, and if $|\tan \gamma'| > \tan 80^\circ$. Then $\text{REF} = 2$ and the reflection begins (described below). If the conditions for reflection are not met $\text{MIT} = 3$, and the program returns to RAYN.

It is desirable to get close to a reflection point in order to accurately define the ray path. But problems occur

if a ray gets too close to a reflection point due to the rapidly increasing ray curvature. This can cause the convergence procedure to fail with a resulting reflection as discussed above. However, very near a reflection point the estimates of ray points become erratic even if the iterations of the ray curvature converge. Accordingly, a reflection is assumed if $DUD > 1$, $|\tan \gamma'| > \tan 89.5^\circ$, and $|\tan \theta'| < \tan 75^\circ$. Then $REF = 3$ and the reflection begins. Otherwise, there is no reflection and a check is made to determine if the new point lies too close to a grid boundary (described below).

A reflection begins with a check of NPT. If $NPT \neq 0$ the ratio of FUD to LI is checked. If the ratio has no remainder page and column headings are written. The printout values of θ , γ , α , and the ray number are calculated. If $MOE = 0$ the ray particulars at the reflection point are written. If $MOE \neq 0$ the English values of the ray particulars are converted to metric values before being written. FUD is increased by one and the value of α is converted back to radians. If $MOE \neq 0$ the metric values of the ray particulars are converted back to English values. Page and column headings are written if the remainder of the ratio of FUD to LI is zero. Then the type of reflection is written. If $REF = 1$ the printed output contains: REFLECTION: SNELLS LAW WITH PHASE VELOCITY. When $REF = 2$ the output is: REFLECTION: PACKET CURVATURE ITERATION NOT CONVERGING. Finally, if $REF = 3$ the output is: REFLECTION: NEAR REFLECTION POINT. After the write statement FUD is increased by one. The previous write statements are omitted if $NPT = 0$.

If $ROP = 0$ the reflection procedure is stopped, $MIT = 6$, and the program returns to RAYN. If $ROP \neq 0$ the ray continues beyond the reflection point. After setting $FLAG2 = 0$ the reflection angles are calculated and the wavelet direction is saved. RCOUNT is increased by one, and if then $RCOUNT > 2$ there has been more than one reflection at the same point. In this case $MIT = 7$ and the program returns to RAYN. If $RCOUNT < 2$, $FLAG1$ is set equal to one and SURFCE is called to calculate the ray particulars based on the reflection angles. Then $FLAG1 = 0$, $FLAGR = 1$, and the values of the ray separation coefficients are saved. This is followed by iterating to the point after reflection using the procedure discussed above.

With the exception of reflection points and if a ray has not been stopped, each new ray point is checked to determine if the point lies within $1\frac{1}{2}$ grid units of a boundary of the water depth grid. If it does $NGO = 2$. Otherwise, NGO remains equal to one.

A number of quantities are saved in case a breakup of the time step interval occurs in HEIGHT. The quantities saved are the coordinates of the previous point, the coordinates of the new point, the previous rotation angle, the present rotation angle, the wave packet direction, the previous wavelet direction, the new wavelet direction, the two previous values of phase speed, and new values of h , v , U , G , and D . Average

values of the wave packet and wavelet directions are calculated for use if the values of p and q are determined at intermediate ray points. The values of the wave packet direction, wavelet direction, and ray curvature are updated, and RCOUNT is initialized to zero.

If REFLCT = 1 the values of x and y are updated and HEIGHT is called. If REFLCT \neq 1 the average values of the packet and wavelet directions are used to determine the values of p and q at the five intermediate points needed for the Runge-Kutta method. In order to keep the wavelet direction from changing on the calls to SURFCE, FLAG1 is set equal to one. After all the values of p and q are computed at the intermediate points SURFCE is called again to reevaluate quantities at the new ray point. Then FLAG1 is set equal to zero, the values of x and y are updated, and HEIGHT is called.

Upon the return to MOVE from HEIGHT, if BRK = 1 there has been a breakup of the time step interval in HEIGHT. Iteration begins for a new point with the new time step interval. If BRK \neq 1 the printout values of both the wave packet and wavelet directions are placed, if necessary, in the range of 0 to 360°. Then, the program returns to RAYN.

3.9 Subroutine HEIGHT. This subroutine is called to calculate the wave height. For the first ray point HEIGHT is called by RAYN. The values of p and q are initialized, and the tolerance for the fourth and fifth order calculations of β and $d\beta/dt$ is calculated. Further, the initial value of $d\beta/dt$ is computed, and the initial values of the friction, refraction, and shoaling coefficients, and the wave height are determined. The program returns to RAYN.

For ray points beyond the first point this subroutine is called by MOVE. The shoaling and friction coefficients are computed. The values of K_F , p , β , and $d\beta/dt$ are saved in case there is a breakup of the time step interval.

The method used in determining β and $d\beta/dt$ depends upon the values of NFK (determined in SURFCE), REFLCT (determined in MOVE), and $|dh/dx'|$ (determined in SURFCE). If NFK = 1 the values of $d\beta/dt$, the difference between the fourth and fifth order solutions of β , and the difference between the fourth and fifth order solutions of $d\beta/dt$ are equated to zero. That is, β is assumed to be constant since the ray is in deep water. The value of β is used to calculate the refraction coefficient.

If NFK = 2 and REFLCT \neq 0 the ray is near a reflection point, and β and $d\beta/dt$ are calculated using the analytical solution for parallel wave speed contours. If FLAGR \neq 0 (determined in MOVE), ROP is put equal to zero; as a result, a ray is not continued beyond a second reflection point should one occur. The refraction coefficient is computed.

If NFK = 2 and REFLCT = 0 the method used to determine

β and $d\beta/dt$ depends upon the value of $|dh/dx'|$. If $|dh/dx'| < 0.00001$, then $d\beta/dt$, the difference between the fourth and fifth order solutions of β , and the difference between the fourth and fifth order solutions of $d\beta/dt$ are equated to zero. That is, β is assumed to be constant since the water depth is taken as invariant. If $|dh/dx'| > 0.00001$ the Runge-Kutta method is used to calculate β and $d\beta/dt$. The fourth order and fifth order solutions as well as the difference of these solutions are determined for both β and $d\beta/dt$. However, before making the calculations, if FLAGR $\neq 0$ the value of ROP is set equal to zero. The refraction coefficient is calculated using the value of β .

After the refraction coefficient is computed several checks are made to determine if the calculations are sufficiently accurate with the time step which was used. Near a reflection point the ray curvature is large; it is necessary to reduce the time step in order to keep successive changes in the wave packet direction small enough to determine an accurate ray path. When computing the ray separation factor using the Runge-Kutta method it is desirable to be able to reduce the time step, if necessary, to keep the truncation error small. Only one of these checks is made at a time. The check to be made is determined by the value of REFLCT. However, regardless of the value of REFLCT, if IFLG $\neq 0$ the value of the breakup time step has previously been determined. In this case the value of INUM is increased by one. The new value of INUM is compared with NUMT to determine if the next ray point should be computed with the breakup time step (described below).

If REFLCT $\neq 0$ and IFLG = 0 the change in the absolute value of the ray direction is checked. If the change is less than 10° the time step is not too large and the value of NUMT is checked (explained below). If the change is greater than or equal to 10° there is a breakup of the time step interval (described below).

If REFLCT = 0 and IFLG = 0 the difference between the Runge-Kutta fourth and fifth order solutions of β and the similar difference of the solutions for $d\beta/dt$ are checked. If the absolute values of both differences are less than BZTOL the time step interval is not divided and NUMT is checked. But, if the absolute value of either difference is greater than or equal to BZTOL there is a breakup of the time step interval and REFRCT = 1. Further, RFRBUM = 1 to determine the format of the printed output of the ray particulars.

If the calculations meet the criteria for accuracy using the assigned time step the value of NUMT is checked. If NUMT > 1 the initial time step interval has been broken. In this case IFLG = 1. The value of INUM is increased by one, and the new value of INUM is compared with NUMT to

determine if the calculations should continue with the breakup time step (described below). When IFLG = 1 further checks for a breakup of the time step interval are not made at new ray points until the breakup ends and calculations are resumed with the initial time step.

When NUMT \leq 1 the time step has its initial value. If $\beta < 0$ there is a focal point or caustic. Then MIT = 4, BRK = 0, and the program returns to MOVE. When $\beta > 0$ the values of p and q at the present point are set equal to the values at the new point. The wave height is computed. If WBCOP = 0 the program returns to MOVE. If WBCOP \neq 0 a test is made to determine if the wave breaks. Then the program returns to MOVE, and if the wave breaks MIT = 5.

The time step interval is halved for a breakup. It is possible for a time step to be halved many times with successive breakups. Thus, it is necessary to place a lower limit on the value of a time step to prevent an inordinate amount of calculations. The new value of the time step is checked, and if it is less than 0.5 sec the ray is stopped. Then MIT = 8, BRK = 0, and the program returns to MOVE.

If the new time step is greater than 0.5 sec the breakup continues. It is necessary to return to the previous ray point. Accordingly, the saved values of G and D are recovered. To determine a new point in MOVE, BRK = 1. The number of intervals, NUMT, the initial time step is divided into is calculated. If at the second ray point the value of the ray curvature is recovered. At other points both the ray curvature and average ray curvature are restored. Further, the values of θ , γ , α , x, y, β , $d\beta/dt$, p, K_F , and the two previous values of v are recovered. The program returns to MOVE.

There are as many ray points in a breakup as required for the travel time to equal the initial time step. Thus, during a breakup, after each new point is determined there is a check to see if the breakup is complete. If INUM < NUMT the breakup is incomplete and the ray is continued with the breakup time step. The value of D is computed and the values of p and q are updated. If there is a focal point or caustic MIT = 4, BRK = 0, and the program returns to MOVE. Otherwise, BRK = 1 and the program returns to MOVE.

When INUM \geq NUMT the breakup ends and calculations resume with the initial time step. The values of IFLG, INUM, and BRK are set equal to zero, and D is computed. There is a check for a focal point or caustic, p and q are updated, and REFRCT and REFLCT are set equal to zero. The wave height is calculated, if WBCOP \neq 0 there is a check to see if the wave breaks, and the program returns to MOVE, as explained above.

3.10 Subroutine SURFCE. SURFCE is called by RAYN and MOVE to calculate h , α , γ , G , p , q , κ_G , and other ray particulars. At the first ray point twelve values of h from CMAT are selected about the point as shown in Figure (2-2). A quadratic surface is fit to the set of water depths. At successive ray points the quadratic surface is determined only if there is a change in the set of twelve water depths. The water depth and its partial derivatives in the fixed xy -system, $\partial h/\partial x$, $\partial h/\partial y$, $\partial^2 h/\partial x^2$, $\partial^2 h/\partial y^2$, and $\partial^2 h/\partial x \partial y$, are determined at the ray point by interpolating on the quadratic surface.

If $h \leq 0$, NDP = 2 and there is a RETURN. If $h > 0$ the ratio of the water depth to the deep water wavelength is computed. If $h/\lambda_d > 0.64$, which defines deep water, NFK = 1. If $h/\lambda_d \leq 0.64$, NFK = 2. VELCTY is called, and after the return if NFK = 1, W = 0. If NFK = 2, CONDER is called to compute W. The values of $\partial v/\partial x$ and $\partial v/\partial y$ are calculated using W.

At each ray point the water depth contours are assumed to be locally parallel, and a $x'y'$ -coordinate system is chosen such that the y' -derivatives vanish. The value of dh/dx' is computed, and if it exceeds 0.00001 the angle α by which the x' -axis is rotated with respect to the x -axis is computed. If $|dh/dx'| \leq 0.00001$ the water depth is assumed to be constant and α remains constant.

If FLAG1 = 0, γ' is computed, and if necessary it is placed within the range $|\gamma'| \leq 360^\circ$. A check is made to determine if there is total reflection. If there is, FLAG2 = 1 and there is a RETURN. Otherwise, FLAG2 = 0 and the new γ' is computed using Snell's law with phase velocity following a set of rules. Using the values of γ' , γ is computed. When FLAG1 \neq 0 these steps for computing the new wavelet direction and the test for total reflection are omitted.

The values of ϕ , G , and dv/dx' are calculated. If NFK = 2, dU/dx' is determined using its unsimplified expression. If NFK \neq 2, the deep water formula is used to calculate dU/dx' . The value of dU/dx' is used in computing dG/dx' .

If NFK \neq 2, the coefficients of the ray separation equation and the ray curvature are set equal to zero. Then there is a RETURN. If NFK = 2, p , $d^2h/(dx')^2$, $d^2v/(dx')^2$, $d^2U/(dx')^2$, $d^2G/(dx')^2$, q , and κ_G are computed. This is followed by a RETURN.

3.11 Subroutine VELCTY. VELCTY is called by SURFCE to calculate v and U . At the first ray point the deep water value of the phase speed and several constants for the ray are computed. If NFK \neq 2 (determined in SURFCE) the deep water value of the phase speed is used, and its value is saved for calculations at the next point. The deep water value of the collinear group speed is calculated, and the

program returns to SURFCE.

If $NFK = 2$ an iterative technique is used to determine v . The iteration continues until the estimate of v differs by less than 0.00005 from an average of the previous estimates of v or until ninety iterations have taken place. The value of v is saved for calculations at the next point, and U is computed using its complete expression. Then the program returns to SURFCE.

3.12 Subroutine CONDER. CONDER is called by SURFCE to compute W . After computing some constants, if $NFK = 1$ (determined in SURFCE) the program returns to SURFCE. If $NFK \neq 1$ the value of W is calculated, then the program returns to SURFCE.

3.13 Subroutine PCD. PCD is called by RAYN. The four water depths in CMAT which are closest to a ray point are compared with the respective depths computed from the twelve point surface fit. The percentage difference of the interpolated water depth to the actual water depth is determined for each grid point, and PCTDIF is the maximum percentage difference of the four values. If the product of the four water depth values in CMAT is zero, PCTDIF = 999.

3.14 Subroutine STORE. STORE is called by RAYN. If $CIN \leq 0$ the x and y coordinates of a ray point are stored in the Ax and Ay arrays, respectively. Then, the program returns to RAYN. If $CIN > 0$ tick marks at travel time intervals equal to CIN are determined along the ray. The travel time is computed using the geometric group speed of the ray. The coordinates of the tick marks are tagged with negative x -values and are stored in sequence with the ray points in the Ax and Ay arrays. The program returns to RAYN.

3.15 Subroutine DRAW. DRAW is called by RAYN to plot each ray. To save plotting time, odd numbered rays begin at their initial points and even numbered rays start at their terminal points. If $FAN = 0$ a ray is numbered at its initial point, otherwise a ray is numbered at its terminal point. If $CIN \leq 0$ a ray has no tick marks. If $CIN > 0$ tick marks are placed on a ray for those positions where the x coordinate is stored in the Ax array with a negative value. The negative values are changed to positive values.

The coordinates for plotting the tick marks depend upon the positions of the tick mark on the ray and the first ray point that is prior to and located more than a specified distance from the tick mark. The separation requirement is necessary since if the two points are too close together numerical inaccuracies in the calculations prevent the tick mark from being perpendicular to the ray. Every tenth tick mark is larger than the others. The program returns to RAYN.

3.16 Listing of the Computer Program.

PROGRAM MAIN (INPUT, OUTPUT, RAYDAT, XYGRID, PLOTT, TAPE5=INPUT	MAIN	1
\$,TAPE6=OUTPUT, TAPE1=RAYDAT, TAPE2=XYGRID)	MAIN	2
C THIS IS A PROGRAM FOR CALCULATING AND PLOTTING THE PATHS OF SURFACE	MAIN	3
C GRAVITY WATER WAVE PACKETS AND FOR CALCULATING THE WAVE HEIGHTS	MAIN	4
C ALONG THESE PATHS CONSIDERING THE EFFECTS OF SHOALING, REFRACTION,	MAIN	5
C AND ENERGY DISSIPATION.	MAIN	6
C	MAIN	7
C THIS PROGRAM WAS COMPLETED NOVEMBER 1977 UNDER A CONTRACT WITH THE	MAIN	8
C GEOGRAPHY PROGRAMS, EARTH SCIENCES DIVISION, THE OFFICE OF NAVAL	MAIN	9
C RESEARCH. THE PROGRAM WAS PREPARED BY	MAIN	10
C J. ERNEST BREEDING, JR., DEPT. OF OCEANOGRAPHY, FLORIDA STATE	MAIN	11
C UNIVERSITY, TALLAHASSEE, FL. 32306	MAIN	12
C K. C. MATSON, NAVAL COASTAL SYSTEMS LAB, PANAMA CITY, FL. 32407	MAIN	13
C NOUROLLOH RIAHI, DEPT. OF OCEANOGRAPHY, FLORIDA STATE UNIVERSITY	MAIN	14
C	MAIN	15
C THIS PROGRAM IS BASED ON A PROGRAM FOR COMPUTING THE PATHS OF	MAIN	16
C MONOCHROMATIC RAYS BY	MAIN	17
C W. STANLEY WILSON, A METHOD FOR CALCULATING AND PLOTTING SURFACE	MAIN	18
C WAVE RAYS. TECHNICAL MEMORANDUM NO. 17, COASTAL ENGINEERING	MAIN	19
C RESEARCH CENTER, 57 PP. (1966) (AD 636 771).	MAIN	20
C WITH THE EXCEPTION OF THE PLOTTING SUBROUTINES, THE WILSON PROGRAM	MAIN	21
C WAS EXTENSIVELY MODIFIED IN ORDER TO COMPUTE THE PATH OF A WAVE	MAIN	22
C PACKET, AND A SUBROUTINE WAS ADDED FOR COMPUTING THE WAVE HEIGHT.	MAIN	23
C	MAIN	24
C THE PROGRAM IS WRITTEN IN FORTRAN IV FOR THE CONTROL DATA CYBER	MAIN	25
C 70 COMPUTER SYSTEMS AND THE GOULD PLOTTER.	MAIN	26
C	MAIN	27
C INPUT PARAMETERS.	MAIN	28
C A,AV ARE, RESPECTIVELY, THE INITIAL DIRECTIONS FROM WHICH THE WAVE	MAIN	29
C PACKET AND WAVELETS COME WITH RESPECT TO TRUE NORTH.	MAIN	30
C CF IS THE FRICTION FACTOR FOR THE FRICTION COEFFICIENT.	MAIN	31
C IF CIN IS NOT ZERO IT IS THE TRAVEL TIME IN SECONDS BETWEEN SUCCESSIVE	MAIN	32
C TICK MARKS ON A RAY.	MAIN	33
C CMAT IS THE WATER DEPTH GRID.	MAIN	34
C CNVPSA IS THE DIRECTION OF THE POSITIVE X-AXIS OF THE WATER DEPTH	MAIN	35
C GRID WITH RESPECT TO TRUE NORTH.	MAIN	36
C IF COL (IS/IS NOT) ZERO THE PLOTTER (WILL NOT/WILL) PAUSE BEFORE	MAIN	37
C A RAY IS PLOTTED.	MAIN	38
C CONTUR SPECIFIES THE SOUNDING DEPTHS IN FEET OR METERS.	MAIN	39
C DATE1, DATE2 DEFINE THE YEAR, MONTH, AND DAY.	MAIN	40
C DCON IS A FACTOR TO CONVERT THE WATER DEPTHS IN CMAT TO FEET	MAIN	41
C OR METERS.	MAIN	42
C DELTAT IS THE TIME STEP IN SECONDS.	MAIN	43
C DIR IS AN IDENTIFIER.	MAIN	44
C EM ARE SURFACE FITTING NUMBERS USED WITH CMAT.	MAIN	45
C IF FAN (IS/IS NOT) ZERO A RAY IS NUMBERED AT ITS (INITIAL/TERMINAL)	MAIN	46
C POINT.	MAIN	47
C GRID IS THE NUMBER OF FEET OR METERS PER GRID UNIT FOR A GIVEN RUN.	MAIN	48
C HGTZ IS THE INITIAL WAVE HEIGHT IN FEET OR METERS.	MAIN	49
C HT IS THE HEIGHT OF THE PLOT IN INCHES OR CENTIMETERS.	MAIN	50
C KRTOL DETERMINES THE ACCURACY IN CALCULATING THE REFRACTION	MAIN	51
C COEFFICIENT.	MAIN	52
C MM,NN ARE THE MAXIMUM X,Y FOR A GIVEN WATER DEPTH GRID.	MAIN	53
C NNSKIP IS THE AMOUNT ADDED TO THE Y-COLUMN IN SELECTING THE NEXT	MAIN	54
C COLUMN FOR LOCATING SOUNDING VALUES.	MAIN	55
C IF MOE (IS/IS NOT) ZERO (ENGLISH/METRIC) UNITS ARE USED.	MAIN	56
C MXPLOT IS THE NUMBER OF PLOTS OR COMPUTER RUNS.	MAIN	57

C IF NAX (IS/IS NOT) ZERO THE AXES OF THE PLOT (WILL NOT/WILL) BE	MAIN	58
C CALIBRATED.	MAIN	59
C IF NCO IS NOT ZERO IT SPECIFIES THE NUMBER OF SOUNDING VALUES FOR	MAIN	60
C A PLOT.	MAIN	61
C NOR IS THE NUMBER OF RAYS FOR A GIVEN RUN.	MAIN	62
C IF NPT IS ZERO PRINTED OUTPUT OCCURS FOR THE INITIAL AND TERMINAL	MAIN	63
C POINTS OF A RAY, AND IF NPT IS NOT ZERO PRINTED OUTPUT OCCURS	MAIN	64
C ADDITIONALLY FOR THOSE POINTS WHICH ARE AN INTEGRAL MULTIPLE	MAIN	65
C OF SK.	MAIN	66
C IF NSH (IS/IS NOT) ZERO THE SHORELINE (IS NOT/IS) DRAWN.	MAIN	67
C IF NXCMAT IS ZERO A WATER DEPTH GRID IS INPUT FOR THE RUN, AND IF	MAIN	68
C NXCMAT IS NOT ZERO THE DEPTH GRID FOR THE PREVIOUS PLOT IS	MAIN	69
C USED AGAIN.	MAIN	70
C PROJECT IS AN IDENTIFIER.	MAIN	71
C IF ROP (IS/IS NOT) ZERO THE RAY (IS NOT/IS) CONTINUED BEYOND A	MAIN	72
C REFLECTION POINT.	MAIN	73
C S ARE SURFACE FITTING NUMBERS USED WITH CMAT.	MAIN	74
C SK. SEE NPT.	MAIN	75
C TT IS THE WAVELET PERIOD IN SECONDS.	MAIN	76
C IF WBCOP (IS/IS NOT) ZERO THE WAVE BREAK TEST (IS NOT/IS) MADE.	MAIN	77
C X,Y ARE THE INITIAL RAY COORDINATES.	MAIN	78
C	MAIN	79
C OUTPUT OF THE RAY PARTICULARS.	MAIN	80
C ALFA IS THE ANGLE OF THE ROTATED XY-SYSTEM (WHERE THE Y-	MAIN	81
C DERIVATIVES VANISH) RELATIVE TO THE WATER DEPTH GRID XY-SYSTEM.	MAIN	82
C D IS THE DISTANCE IN GRID UNITS BETWEEN SUCCESSIVE RAY POINTS.	MAIN	83
C D(BETA)/DT IS THE INITIAL VALUE OF THE TIME DERIVATIVE OF THE	MAIN	84
C RAY SEPARATION FACTOR.	MAIN	85
C FK IS THE RAY CURVATURE OF THE PACKET IN RADIANS/GRID UNIT.	MAIN	86
C $G = U \cos(\text{PACK} - \text{WAVE})$ IS THE GEOMETRIC GROUP SPEED IN	MAIN	87
C FEET/SECOND OR METERS/SECOND.	MAIN	88
C H IS THE WATER DEPTH IN FEET OR METERS.	MAIN	89
C HGT IS THE WAVE HEIGHT IN FEET OR METERS.	MAIN	90
C KF IS THE FRICTION COEFFICIENT.	MAIN	91
C KR IS THE REFRACTION COEFFICIENT.	MAIN	92
C KS IS THE SHOALING COEFFICIENT.	MAIN	93
C MAX IS AN INDEX TO NUMBER POINTS ALONG A RAY.	MAIN	94
C NO IS THE NUMBER OF INTERVALS THE INPUT TIME STEP IS DIVIDED INTO.	MAIN	95
C PACK IS THE DIRECTION FROM WHICH THE WAVE PACKET (RAY) COMES.	MAIN	96
C PCTDIF IS THE MAXIMUM OF THE PERCENTAGE DIFFERENCES AT THE 4 GRID	MAIN	97
C POINTS CLOSEST TO THE RAY POINT OF THE SURFACE FIT DERIVED WATER	MAIN	98
C DEPTH RELATIVE TO THE ACTUAL DEPTH.	MAIN	99
C U IS THE COLLINEAR GROUP SPEED IN FEET/SECOND OR METERS/SECOND.	MAIN	100
C V IS THE PHASE SPEED IN FEET/SECOND OR METERS/SECOND.	MAIN	101
C WAVE IS THE DIRECTION FROM WHICH THE WAVELETS (IN A PACKET) COME.	MAIN	102
C X,Y ARE THE COORDINATES OF A RAY POINT.	MAIN	103
C DIMENSION S(6,6),EH(6,12),C(12),YVW(6),E(6)	MAIN	104
C \$,CMAT(100,100),AX(200),AY(200),CONTUR(9)	MAIN	105
C REAL KR,KF,KS,KRTOL,KFC	MAIN	106
C INTEGER DX1,DX2,ROP,WBCOP,FAN,COL,FUD,BRK,SK,FLAGR,FLAG3	MAIN	107
C \$,REFLECT,REFLUM,REFRCT,REFRUM	MAIN	108
C COMMON S,EH,E,YVW,CMAT,C,AX,AY,CONTUR,PROJECT,GRID,DCON,FAN,DATE1	MAIN	109
C \$,DATE2,CIN,DIR,ROP,TT,WBCOP,MOE,DY,DELTA1,SOLTA1,D,HGT,HGTZ,SVX	MAIN	110
C \$,SVY,SDEP,H,DEP,HL,V,SAVV,PPEV,SPREV,U,SAVU,GZERO,G,SG,SVG,CUD,KS	MAIN	111
C \$,DGDY,SVX,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX	MAIN	112
C \$,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUP,IFLG,PCOUNT,AMP,ANN	MAIN	113
C \$,REFLECT,REFLUM,REFRCT,REFRUM,BRK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZ	MAIN	114
C \$,SBZ,BDZ,SBLZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5	MAIN	115

C THE MAIN PROGRAM IS USED TO READ THE INPUT DATA, TO CONTROL THE	MAIN	116
C CALCULATIONS FOR EACH RAY, AND TO PREPARE THE PLOTS. SUBROUTINES	MAIN	117
C TITLE, NUMCON, SHORE, PLOT, AND RAYN ARE CALLED.	MAIN	118
CALL PLOTS (10.,0.,5HPLOTT,120)	MAIN	119
MMAX=2000 & LI=6J & LII=(LI-4)/3 & COFI=12.	MAIN	120
READ (1,1) ((S(OX2,OX1),OX2=1,6),OX1=1,6)	MAIN	121
1 FOPMAT (8F10.7)	MAIN	122
READ (1,2) ((E(OX1,OX2),OX2=1,12),OX1=1,6)	MAIN	123
2 FOPMAT (36F2.0)	MAIN	124
READ (1,3) (MXPLOT,PROJECT,DATE1,DATE2,DIR)	MAIN	125
3 FOPMAT (I2,1X,3(A6,1X),A6)	MAIN	126
DO 399 NPLOT=1,MXPLOT	MAIN	127
READ (1,4) (NOR,NPT,SK,HT,CIN,NAX,NSH,NCO,NXCMAT)	MAIN	128
4 FOPMAT (3(2X,I3),5X,2(F6.3,2X),4(2X,I3))	MAIN	129
READ (1,5) (MM,NN,CNVRSA,GRID,DCON,MOE,NNSKIP)	MAIN	130
5 FOPMAT (2(2X,I3),1X,F7.3,2(1X,F9.3),4X,I3,2X,I3)	MAIN	131
IF (MOE.EQ.0) GO TO 2J	MAIN	132
C CONVERT TO ENGLISH UNITS FOR CALCULATIONS	MAIN	133
HT=HT/2.54 & DCON=DCON/0.3048 & GRID=GRID/0.3048	MAIN	134
20 CIN=CIN/3600. & AMM=MM-1. & ANN=NN-1.	MAIN	135
DY=ANN/HT & SCLI=GRID*DY*COFI	MAIN	136
CALL TITLE (NPLOT,NAX,SCLI,HT)	MAIN	137
IF (NXCMAT.NE.0) GO TO 3939	MAIN	138
READ (2,11) ((C*AT(J,I),I=1,4M),J=1,NN)	MAIN	139
11 FOPMAT (16F5.0)	MAIN	140
3939 IF (NCO.LE.0) GO TO 493	MAIN	141
READ (1,495) (CONTR(I),I=1,NCO)	MAIN	142
495 FOPMAT (9F8.2)	MAIN	143
CALL NUMCON (MM,NN,NCO,NNSKIP)	MAIN	144
493 IF (NSH.EQ.0) GO TO 3937	MAIN	145
CALL SHORE (MM,NN)	MAIN	146
3937 DO 15 N=1,NOR	MAIN	147
READ (1,6) (DELTAT,TT,X,Y,A,AV,HGTZ,CF,KRTOL,FOP,WBCOP,FAN,COL)	MAIN	148
6 FOPMAT (7(F6.2,2X),2(F6.4,2X),4(I1,1X))	MAIN	149
IF (MOE.EQ.0) GO TO 22	MAIN	150
HGTZ=HGTZ/0.3048	MAIN	151
22 SOLTAT=DELTAT & WL=32.2*(TT**2)/6.2831854	MAIN	152
A=CNVRS-A+180. & AV=CNVRS-AV+180. & MAXQ=1 & FUD=0.	MAIN	153
BRK=0. & REFLCT=0. & RFLBUM=0. & REFRCT=0. & RFRBUM=0.	MAIN	154
FLAGR=0. & FLAG3=1. & IFLG=0 & ALFA=0.	MAIN	155
IF (COL.EQ.0) GO TO 4321	MAIN	156
CALL PLOT (3.,0.4,-3)	MAIN	157
4321 A=.0174532925 & AV=AV*.0174532925	MAIN	158
CALL RAYN(X,Y,A,NPLOT,N,PMAX,LII,NPT,LII,AV)	MAIN	159
15 CONTINUE	MAIN	160
CALL PLOT (-3.,-.4,-3)	MAIN	161
CALL PLOT (0.,0.,999)	MAIN	162
399 CONTINUE	MAIN	163
WRITE (6,9999)	MAIN	164
9999 FOPMAT (1H1,17H THIS IS THE END.)	MAIN	165
CALL EXIT	MAIN	166
END	MAIN	167

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SUBROUTINE TITLE (NPLOT,NAX,SCLI,HT)	TITLE	1
DIMENSION S(6,6),EM(6,12),C(12),YVM(6),E(6)	TITLE	2
\$,CMAT(100,100),AX(200),AY(200),CONTUR(9)	TITLE	3
REAL KR,KF,KS,KRTOL,KFC	TITLE	4
COMMON S,EM,E,YVM,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1	TITLE	5
\$,DATE2,CIN,DIR,ROP,TT,HSCOP,MOE,DY,DELTAT,SOLTAT,D,HGT,HGTZ,SVX	TITLE	6
\$,SVY,SDEP,W,DEP,HL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUO,KS	TITLE	7
\$,DGOX,SVA,TPI,SAV,SAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHOX	TITLE	8
\$,SVFKB,SAVFK,FKBAR,MAXG,SK,FUD,NUMT,INUM,IFLG,RCOUNT,AMM,ANN	TITLE	9
\$,REFLECT,RFLEUM,REFRCT,XFRBUM,BFK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZ	TITLE	10
\$,SEZ,BDZ,SOLZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5	TITLE	11
C IN THIS SUBROUTINE THE PLOT IS LABELED AND STRAIGHT-LINE BORDERS	TITLE	12
C ARE DRAWN. SUBROUTINES SYMBOL, NUMBER, PLOT, AND AXIS2 ARE CALLED.	TITLE	13
CALL PLOT(3.,0.4,3)	TITLE	14
RT=AMM/DY \$ XNPLOT=NPLOT	TITLE	15
C DRAW LABELS FOR PLOT	TITLE	16
CALL SYMBOL(1.25,0.4,.2,17HPRCJ. NO. ,90.,17)	TITLE	17
CALL SYMBOL(1.25,2.4,.2,PROJCT,90.,6)	TITLE	18
CALL SYMBOL(1.25,4.0,.2,DATE1,90.,6)	TITLE	19
CALL SYMBOL(1.25,5.2,.2,DATE2,90.,2)	TITLE	20
CALL SYMBOL(1.50,0.4,.2,23HSC = 1/ , CIN =,90.,23)	TITLE	21
CALL NUMBER(1.50,2.0,.2,SCLI,90.,-1)	TITLE	22
CALL NUMBER(1.50,5.2,.2,CIN*3600.,90.,-1)	TITLE	23
CALL SYMBOL(1.75,0.4,.2,19HPLOT NO. , DIR. =,90.,19)	TITLE	24
CALL NUMBER(1.75,2.2,.2,XNPLOT,90.,-1)	TITLE	25
CALL SYMBOL(1.75,4.4,.2,DIR,90.,6)	TITLE	26
IF (NAX .NE. 0) GO TO 705	TITLE	27
C DRAW STRAIGHT-LINE BORDERS FOR PLOT	TITLE	28
CALL PLOT(3.,0.4,3)	TITLE	29
CALL PLCT(3.,HT+.4,2)	TITLE	30
GO TO 706	TITLE	31
705 CALL AXIS2(3.,0.4,1HY,1,HT,90.,0.,DY)	TITLE	32
CALL AXIS2(3.,.4,1HX,-1,RT,0.,0.,DY)	TITLE	33
CALL PLCT(3.,HT+.4,3)	TITLE	34
706 CALL PLCT(RT*3.,HT+.4,2)	TITLE	35
CALL PLCT(RT*3.,.4,2)	TITLE	36
IF (NAX .NE. 0) GO TO 707	TITLE	37
CALL PLCT(3.,0.4,2)	TITLE	38
707 CALL PLOT(3.,0.4,-3)	TITLE	39
YHT=HT	TITLE	40
RETURN	TITLE	41
END	TITLE	42

SUBROUTINE AXIS2(X,Y,BCD,NC,SIZE,THETA,YMIN,DY)	AXIS2	1
DIMENSION BCD(16)	AXIS2	2
C IN THIS SUBROUTINE THE AXES ARE DRAWN, CALIBRATED, AND LABELED.	AXIS2	3
C SUBROUTINES PLOT, NUMBER, AND SYMBOL, ARE CALLED.	AXIS2	4
BIGN=1.0	AXIS2	5
IF (NC .GE. 0) GO TO 2	AXIS2	6
BIGN=-1.0	AXIS2	7
2 NAC=IABS(NC) \$ TH=THETA*.017453294	AXIS2	8
N=DY*SIZE+.5 \$ CTH=COS(TH) \$ STH=SIN(TH) \$ TN=N	AXIS2	9
X3=X \$ Y3=Y \$ XA=X-.0.1*BIGN*STH \$ YA=Y+.0.1*BIGN*CTH	AXIS2	10

C DRAW AXIS WITH CALIBRATED TICK MARKS	AXIS2	11
CALL PLOT(XA,YA,3)	AXIS2	12
DO 20 I=1,N	AXIS2	13
CALL PLOT(XB,YB,2)	AXIS2	14
XC=XB+CTH/DY & YC=YB+STH/DY	AXIS2	15
CALL PLOT(XC,YC,2)	AXIS2	16
XA=XA+CTH/DY & YA=YA+STH/DY	AXIS2	17
CALL PLOT(XA,YA,2)	AXIS2	18
XB=XC & YB=YC	AXIS2	19
20 CONTINUE	AXIS2	20
ABSV=YMIN+TN & XA=XB-((.20*BIGN-.05)*STH-.02857*CTH	AXIS2	21
YA=YE+((.20*BIGN-.05)*CTH-.02857*STH & N=N+1	AXIS2	22
C NUMBER THE ORIGIN AND EVERY FIFTH TICK MARK	AXIS2	23
DO 30 I=1,N	AXIS2	24
IF (AMOD(ABSV,5.) .NE. 0.) GO TO 100	AXIS2	25
CALL NUMBER(XA,YA,.1,ABSV,THETA,-1)	AXIS2	26
100 ABSV=ABSV-1. & XA=XA-CTH/DY & YA=YA-STH/DY	AXIS2	27
30 CONTINUE	AXIS2	28
C LABEL THE AXIS	AXIS2	29
TNC=NAC*7 & XA=X+(SIZE/2.0-.06*TNC)*CTH-((.07*BIGN*.36)*STH	AXIS2	30
YA=Y+(SIZE/2.0-.06*TNC)*STH-((.07*BIGN*.36)*CTH	AXIS2	31
CALL SYMBOL(XA,YA,.14,900,THETA,NAC)	AXIS2	32
RETURN	AXIS2	33
END	AXIS2	34
 SUBROUTINE NUMCON (MM,NN,NCO,NNSKIP)	NUMCON	1
DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)	NUMCON	2
F,CMAT(100,100),AX(200),AY(2000),CONTR(9)	NUMCON	3
REAL KP,KF,KS,KRTOL,KFC	NUMCON	4
COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTR,PROJECT,GRID,CON,FAN,DATE1	NUMCON	5
F,DATE2,CIN,DIR,POP,TT,HCOP,MOE,OY,DELTAT,SDLTAT,U,HGT,HGTZ,SVX	NUMCON	6
S,SVY,SDEP,M,DEP,HL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVS,DUD,KS	NUMCON	7
S,DGDX,SVA,TPI,SAV,SAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX	NUMCON	8
S,SVFKB,SAVFK,FKBA,KAXO,SK,FUD,NUMT,INUP,IFLG,PCOUNT,AM,ANN	NUMCON	9
S,REFCT,REFLEUP,REFRCT,REFRUP,PFK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZ,NUMCON	NUMCON	10
S,SCZ,BOZ,SBZ,KRTOL,KP,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5	NUMCON	11
C IN THIS SUBROUTINE SPECIFIED SOUNDING DEPTHS ARE LOCATED AND	NUMCON	12
C DRAWN ON THE PLOT. SUBROUTINES NUMBER AND PLOT ARE CALLED.	NUMCON	13
NOD=NN-1 & MODD=M-1	NUMCON	14
IF (MOE .EQ. 0) GO TO 2	NUMCON	15
C CONVERT TO ENGLISH UNITS FOR CALCULATIONS	NUMCON	16
DO 7000 KC=1, NCO	NUMCON	17
CONTR(KC)=CONTR(KC)/0.3048	NUMCON	18
7000 CONTINUE	NUMCON	19
C SELECT Y-COLUMN	NUMCON	20
2 DO 5000 J=2, NOD, NNSKIP	NUMCON	21
YJ=J-1 & KKK=1	NUMCON	22
C SELECT SOUNDING DEPTH	NUMCON	23
DO 8000 KC=1,NCO	NUMCON	24
KWIT=J & NDIF=3 & I=M-1	NUMCON	25
C SEARCH COLUMN FOR THE GIVEN SOUNDING DEPTH	NUMCON	26
DO 1010 II=1,MODD	NUMCON	27
XI=I-1 & IL=I+1 & XL=IL-1	NUMCON	28
IF (KWIT .GT. 0) GO TO 8000	NUMCON	29
IF (CMAT(J,I) .GT. 0) GO TO 20	NUMCON	30
KWIT=1	NUMCON	31
20 IF (CMAT(J,I)*GCON-CONTR(KC)) 12,11,13	NUMCON	32
11 AX(KKK)=XI & AY(KKK)=CONTR(KC) & KKK=KKK+1 & NDIF=3	NUMCON	33
GO TO 1010	NUMCON	34
12 GO TO (14,77,14),NDIF	NUMCON	35
14 NDIF=1	NUMCON	36
GO TO 1010	NUMCON	37
13 GO TO (77,15,15),NDIF	NUMCON	38
15 NDIF=2	NUMCON	39
GO TO 1010	NUMCON	40

C LINEARLY INTERPOLATE FOR THE SOUNDING DEPTH	NUMCON	41
77 SLPX=(DCON*(CMAT(J,IL)-CMAT(J,I)))/(XL-XI)	NUMCON	42
XP=(CONTUR(KC)-DCON*CMAT(J,I))/SLPX*XI	NUMCON	43
AX(KKK)=XP & AY(KKK)=CONTUR(KC) & KKK=KKK+1	NUMCON	44
GO TO (81,82),NDIF	NUMCON	45
81 NDIF=2	NUMCON	46
GO TO 1010	NUMCON	47
82 NDIF=1	NUMCON	48
1010 I=I-1	NUMCON	49
8000 CONTINUE	NUMCON	50
C DRAW OUT SOUNDING DEPTHS FOR EACH SELECTED Y-COLUMN	NUMCON	51
KKK=KKK-1	NUMCON	52
IF (KKK-1) 5000,668,670	NUMCON	53
670 KKL=KKK-1	NUMCON	54
DO 997 IA=1,KKL	NUMCON	55
IAD=IA+1	NUMCON	56
DO 997 IB=IAD,KKK	NUMCON	57
IF (AX(IA) .LE. AX(IB)) GO TO 997	NUMCON	58
XMIN=AX(IA) & AX(IA)=AX(IB) & AX(IB)=XMIN	NUMCON	59
XMIN=AY(IA) & AY(IA)=AY(IB) & AY(IB)=XMIN	NUMCON	60
997 CONTINUE	NUMCON	61
668 IF (MOD(J,2) .NE. 0) GO TO 104	NUMCON	62
KONE=KKK & KADD=-1 & LAST=1	NUMCON	63
GO TO 105	NUMCON	64
104 KONE=1 & KADD=1 & LAST=KKK	NUMCON	65
105 IF (MOE .EQ. 0) GO TO 4	NUMCON	66
C CONVERT SOUNDING DEPTH TO METRIC UNITS BEFORE DRAWING ON PLOT	NUMCON	67
AY(KONE)=AY(KONE)*0.3048	NUMCON	68
4 CALL NUMBER(AX(KONE)/DY,YJ/DY,0.1,AY(KONE),J,C,-1)	NUMCON	69
IF (KONE .EQ. LAST) GO TO 5000	NUMCON	70
KONE=KONE+KADD	NUMCON	71
GO TO 105	NUMCON	72
5000 CONTINUE	NUMCON	73
CALL PLOT(0.,0.,-3)	NUMCON	74
RETURN	NUMCON	75
END	NUMCON	76
 SUBROUTINE SHORE(MM,N4)	SHORE	1
DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)	SHORE	2
&,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)	SHORE	3
REAL KF,KF,KS,KRTOL,KFC	SHORE	4
COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJECT,GRID,DCON,FAN,DATE1	SHORE	5
&,DATE2,CIN,CIR,POP,TT,HCOP,MOE,DY,DELTAT,SULTAT,D,HGT,HGTZ,SVX	SHORE	6
&,SVY,SDEP,W,DEP,HL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,S,SVS,DUJ,KS	SHORE	7
&,DGOX,SA,TPI,SAV,SAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX	SHORE	8
&,SVFKP,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,RCOUNT,AMP,ANN	SHORE	9
&,REFLOT,REFLBU,REFRCT,REFRBU,BFK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZSHORE	SHORE	10
&,SBZ,SDZ,SDLZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5	SHORE	11
C IN THIS SUBROUTINE THE SHORELINE IS DRAWN. SUBROUTINE PLOT IS USED.	SHORE	12
PONT(X1,X2,D1,D2)=X1-D1*((X1-X2)/(D1-D2))	SHORE	13
IC=3	SHORE	14
C SELECT Y-COLUMN	SHORE	15
DO 1 J=1,NN	SHORE	16
YJ=J-1 & JL=J-1 & YL=JL-1 & I=MM	SHORE	17
C SEARCH COLUMN FOR ZERO WATER DEPTH STARTING WITH MAXIMUM X	SHORE	18
DO 2 II=1,MM	SHORE	19
XI=I-1 & IL=I+1 & XL=IL-1	SHORE	20
IF (CMAT(J,I)) 100,200,300	SHORE	21

100	IF (IC .GT. 2) GO TO 102	SHORE	22
	C LINEARLY INTERPOLATE FOR ZERO WATER DEPTH	SHORE	23
101	XP=PONT(XI,XL,CMAT(J,I),CMAT(J,IL))	SHORE	24
	CALL PLOT(XP/DY,YJ/DY,IC)	SHORE	25
	IC=2	SHORE	26
	GO TO 1	SHORE	27
102	IF (J .LE. 1) GO TO 101	SHORE	28
	YP=PONT(YJ,YL,CMAT(J,1),CMAT(JL,1))	SHORE	29
	CALL PLOT (C,J,YP/DY,IC)	SHORE	30
	IC=2	SHORE	31
	XP=PONT(XI,XL,CMAT(J,I),CMAT(J,IL))	SHORE	32
	CALL PLOT(XP/DY,YJ/DY,IC)	SHORE	33
	GO TO 1	SHORE	34
200	IF (II .NE. MM) GO TO 201	SHORE	35
	CALL PLOT(XI/DY,YJ/DY,IC)	SHORE	36
	IF (IC .GT. 2) GO TO 204	SHORE	37
	IC=3	SHORE	38
	GO TO 1	SHORE	39
204	IC=2	SHORE	40
	GO TO 1	SHORE	41
201	IF (IC .LE. 2) GO TO 207	SHORE	42
	IF (J .LE. 1) GO TO 207	SHORE	43
	YP=PONT(YJ,YL,CMAT(J,1),CMAT(JL,1))	SHORE	44
	CALL PLOT (C,J,YP/DY,IC)	SHORE	45
	IC=2	SHORE	46
207	CALL PLOT(XI/DY,YJ/DY,IC)	SHORE	47
	IC=2	SHORE	48
	GO TO 1	SHORE	49
300	IF (II .NE. MM) GO TO 2	SHORE	50
	IF (IC .GT. 2) GO TO 1	SHORE	51
	YP=PONT(YJ,YL,CMAT(J,1),CMAT(JL,1))	SHORE	52
	CALL PLOT (C,J,YP/DY,IC)	SHORE	53
	IC=3	SHORE	54
	GO TO 1	SHORE	55
2	I=I-1	SHORE	56
1	CONTINUE	SHORE	57
	CALL PLOT(J.,C.,-3)	SHORE	58
	RETURN	SHORE	59
	END	SHORE	60

SUBROUTINE RAYN(X,Y,A,NPLOT,N,MMAX,LI,NPT,LII,AV)	RAYN	1
DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)	RAYN	2
\$,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)	RAYN	3
REAL KP,KF,KS,KRTOL,KFC	RAYN	4
INTEGER DX1,DX2,SK,FUJ,RCOUNT,FLAG1,RFLBUM,RFRBUM	RAYN	5
COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJECT,GRID,DCON,FAN,DATE1	RAYN	6
\$,DATE2,CIN,DIR,ROP,TT,W3COP,MCE,DY,DELTAT,SDLTAT,D,HGT,HGTZ,SVX	RAYN	7
\$,SVY,SDEP,W,DEP,HL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS	RAYN	8
\$,OGDX,SAV,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX	RAYN	9

	\$,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUM,IFLG,RCOUNT,APM,ANN	RAYN	10
	\$,REFLECT,RFLPUM,REFRCT,RFRBUM,BRK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZRAYN	RAYN	11
	\$,SEZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,GOT,Q1,Q2,Q3,Q4,Q5	RAYN	12
C	THIS SUBROUTINE IS USED TO CONTROL THE CALCULATION OF THE WAVE	RAYN	13
C	PACKET PARTICULARS, MOST OF THE PRINTED OUTPUT, AND THE PLOTS OF	RAYN	14
C	THE WAVE PACKET PATHS. SUBROUTINES SURFCE, MOVE, HEIGHT,	RAYN	15
C	PCD, STORE, AND DRAW ARE CALLED.	RAYN	16
	NDP=1 \$ NFK=1 \$ NGO=1 \$ KREST=G \$ KCIN=0 \$ RCOUNT=0. \$ FLAG1=1.	RAYN	17
	CALL SURFCE(X,Y,A,FK,NFK,NDP,AV)	RAYN	18
	FLAG1=0. \$ INUM=0 \$ SVALFA=ALFA \$ SAV=AV \$ SDEP=DEP	RAYN	19
	SAVV=V \$ PREV=SAVV \$ SAVU=U \$ SG=G \$ GZERO=G	RAYN	20
	CALL MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP,AV,LI,NPT)	RAYN	21
	CALL HEIGHT(X,Y,A,FK,NGO,MIT,NFK,NDP,AV)	RAYN	22
	TIMEQ=0. \$ ANGLE=A*57.29577951 \$ ANGLE=CNVRSR-ANGLE+180.	RAYN	23
	GAM=AV*57.29577951 \$ GAM=CNVRSR-GAM*180.	RAYN	24
	IF (NPT .NE. 0) GO TO 150	RAYN	25
	IF (N .LE. 1) GO TO 800	RAYN	26
	IF (MOD(N,LI) .NE. 0) GO TO 803	RAYN	27
C	WRITE PAGE AND COLUMN HEADINGS	RAYN	28
800	WRITE (6,8) (PROJECT,DATE1,DATE2,NPLOT)	RAYN	29
8	FORMAT (1H,11HPROJECT NO.,A6,1H,,2X,2A6,1H,,5X,8HPLOT NO.,I3,/))	RAYN	30
	IF (MOE .NE. 0) GO TO 455	RAYN	31
	WRITE (6,450)	RAYN	32
450	FORMAT (1X,52HTHE OUTPUT IS IN ENGLISH UNITS. H,HGT(FEET). G,U,V,VRAYN	RAYN	33
	\$14H(FEET/SECOND),,/))	RAYN	34
	GO TO 452	RAYN	35
455	WRITE (6,451)	RAYN	36
451	FORMAT (1X,52HTHE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V,VRAYN	RAYN	37
	\$15H(METER/SECOND),,/))	RAYN	38
452	WRITE (6,851)	RAYN	39
851	FORMAT (1X,1HN,2X,6HPERIOD,1X,3HMAX,2X,1HX,6X,1HY,6X,1HH,7X,+HPACKRAYN	RAYN	40
	\$,3X,4HWAVE,3X,3HHT,5X,6HDELTAT,1X,2HCF,7X,5HKRTOL,/))	RAYN	41
803	ALFA=ALFA*57.29577951	RAYN	42
	IF (MOE .EQ. 0) GO TO 210	RAYN	43
	DEP=DEP*0.3048 \$ G=G*0.3048 \$ U=U*0.3048	RAYN	44
	V=V*0.3048 \$ HGT=HGT*0.3048	RAYN	45
C	WRITE WAV PARTICULARS FOR THE INITIAL POINT	RAYN	46
210	WRITE (6,853) (N,IT,MAXQ,X,Y,DEP,ANGLE,GAM,HGT,DELTAT,CF,KRTOL)	RAYN	47
853	FORMAT (1X,I3,F6.1,1X,I5,2F7.2,F8.2,2F7.2,F8.4,F6.2,2(1X,F8.6))	RAYN	48
	GO TO 522	RAYN	49
3	MAXQ=1+MAXQ	RAYN	50
	IF (MAXQ>KCIN .LT. MMA) GO TO 399	RAYN	51
	WRITE (6,401)	RAYN	52
401	FORMAT (80X,35HDIMENSION OF OUTPUT-ARRAYS EXCEEDED)	RAYN	53
	GO TO 15	RAYN	54
399	ZCXY=G	RAYN	55
	CALL MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP,AV,LI,NPT)	RAYN	56
	IF (NDP .EQ. 1) GO TO 396	RAYN	57

402	WRITE (6,403)	RAYN	58
403	FORMAT (80X,17HRAY REACHED SHORE)	RAYN	59
	MAXQ=MAXQ-1	RAYN	60
	GO TO 15	RAYN	61
396	GO TO (397,397,404,514,515,525,516,528),MIT	RAYN	62
404	WRITE (6,405)	RAYN	63
405	FORMAT (80X,41HPACKET CURVATURE ITERATION NOT CONVERGING)	RAYN	64
	MAXQ=MAXQ-1	RAYN	65
	GO TO 15	RAYN	66
514	WRITE (6,504)	RAYN	67
504	FORMAT (80X,22HCAUSTIC OR FOCAL POINT)	RAYN	68
	MAXQ=MAXQ-1	RAYN	69
	GO TO 15	RAYN	70
515	WRITE (6,505)	RAYN	71
505	FORMAT (80X,11HWAVE BREAKS)	RAYN	72
	MAXQ=MAXQ-1	RAYN	73
	GO TO 15	RAYN	74
525	IF (NPT .NE. 0) GO TO 527	RAYN	75
	WRITE (6,526)	RAYN	76
526	FORMAT (80X,10HREFLECTION)	RAYN	77
527	MAXQ=MAXQ-1	RAYN	78
	GO TO 15	RAYN	79
516	WRITE (6,517)	RAYN	80
517	FORMAT (80X,18HREFLECTION HANG-UP)	RAYN	81
	MAXQ=MAXQ-1	RAYN	82
	GO TO 15	RAYN	83
528	WRITE (6,529)	RAYN	84
529	FORMAT (80X,36HBREAKUP TIME STEP LESS THAN 0.5 SECOND)	RAYN	85
	MAXQ=MAXQ-1	RAYN	86
	GO TO 15	RAYN	87
397	TIMEQ=TIMEQ+(D*GRID/(1800.*(G+ZCXY))) \$ PALFA=ALFA*57.29577951	RAYN	88
	ANGLE=A*57.29577951 \$ ANGLE=CNVRS-ANGLE+180. \$ PFK=FK	RAYN	89
	PDEP=DEP \$ PD=D \$ PG=G \$ PV=V \$ PU=U \$ PHGT=HGT \$ PKS=KS	RAYN	90
	PKF=KFC \$ PKR=KF \$ GAM=AV*57.29577951 \$ GAM=CNVRS-GAM+180.	RAYN	91
	IF (NPT .EQ. 0) GO TO 161	RAYN	92
160	CALL PCD(C,E,PCTDIF)	RAYN	93
	IF (MAXQ .EQ. 1 .OR. 403(MAXQ,SK) .EQ. 0) GO TO 3041	RAYN	94
	GO TO 161	RAYN	95
C	WRITE RAY PARTICULARS FOR SELECTED RAY POINTS	RAYN	96
3041	ALFA=ALFA*57.29577951	RAYN	97
	IF (MOE .EQ. 0) GO TO 200	RAYN	98
	DEP=DEP*0.3048 \$ G=G*0.3048 \$ U=U*0.3048	RAYN	99
	V=V*0.3048 \$ HGT=HGT*0.3048	RAYN	100
200	IF (MOD(FUD,LI) .NE. 0) GO TO 3043	RAYN	101
C	WRITE PAGE AND COLUMN HEADINGS	RAYN	102
	WRITE (6,7) (PROJECT,DATE1,DATE2,NPLOT,IT,N,DELTAT,CF,KRTOL)	RAYN	103
7	FORMAT (1H1,11HPROJECT NO.,A6,1H,,2X,2A6,1H,,5X,8HPLOT NO.,I3,1H,,	RAYN	104
	\$1X,7HPERIOD=,F5.1,4HSEC.,1H,,1X,7HPAY NO.,I3,1H,,1X,7HDELTAT=,	RAYN	105
	\$F6.2,1H,,1X,3HCF=,F8.6,1H,,1X,6HKRTOL=,F8.6,//)	RAYN	106
	IF (MAXQ .NE. 1) GO TO 453	RAYN	107
	IF (MOE .NE. 0) GO TO 465	RAYN	108
	WRITE (6,470) 6DZ	RAYN	109
470	FORMAT (1X,52HTHE OUTPUT IS IN ENGLISH UNITS. H,HGT(FEET). G,U,V,RAYN	RAYN	110
	\$14H(FEET/SECONO).,32X,13HD(BETA)/DT = ,E10.3,//)	RAYN	111
	GO TO 453	RAYN	112
465	WRITE (6,471) 8DZ	RAYN	113
471	FORMAT (1X,52HTHE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V,RAYN	RAYN	114
	\$15H(METER/SECONO).,31X,13HD(BETA)/DT = ,E10.3,//)	RAYN	115

453	WRITE (6,150)	RAYN	116
150	FORMAT (1X,3HMAX,1X,1HX,6X,1HY,6X,1HH,7X,4HPACK,3X,4HNAVE,3X,1HD,	RAYN	117
	8X,2HFK,8X,4HALFA,5X,1HG,5X,1HU,5X,	RAYN	118
	81HV,5X,3HHGT,5X,2HKS,6X,2HKF,7X,2HNO,1X,2HKR,7X,6HPCTDIF,/))	RAYN	119
3043	FUD=FUD+1.	RAYN	120
	IF (RFLBUM .NE. 0) GO TO 518	RAYN	121
	IF (PFREUM .NE. 0) GO TO 520	RAYN	122
	WRITE (6,612) (MAXQ,X,Y,DEP,ANGLE,GAM,D,FK,ALFA,G,U,V,HGT,	RAYN	123
	8KS,KFC,KR,PCDIF)	RAYN	124
612	FORMAT (1X,I4,2F7.2,F3.2,2F7.2,2E10.2,F8.2,1X,3F6.2,	RAYN	125
	83F8.4,4X,F9.4,F7.2)	RAYN	126
	GO TO 522	RAYN	127
518	RFLBUM=RFRBUM=0	RAYN	128
C	USE FORMAT FOR REFLECTION BREAK UP OF TIME STEP INTERVAL	RAYN	129
	WRITE (6,613) (MAXQ,X,Y,DEP,ANGLE,GAM,D,FK,ALFA,G,U,V,HGT,	RAYN	130
	8KS,KFC,NUMT,KR,PCDIF)	RAYN	131
613	FORMAT (1X,I4,2F7.2,F3.2,2F7.2,E10.2,1H*,E9.2,F8.2,1X,3F6.2,	RAYN	132
	83F8.4,1X,I3,F9.4,F7.2)	RAYN	133
	GO TO 522	RAYN	134
520	RFRBUM=0.	RAYN	135
C	USE FORMAT FOR REFRACTION (BETA) BREAK UP OF TIME STEP INTERVAL	RAYN	136
	WRITE (6,614) (MAXQ,X,Y,DEP,ANGLE,GAM,D,FK,ALFA,G,U,V,HGT,	RAYN	137
	8KS,KFC,NUMT,KR,PCDIF)	RAYN	138
614	FORMAT (1X,I4,2F7.2,F3.2,2F7.2,2E10.2,F8.2,1X,3F6.2,	RAYN	139
	83F8.4,1X,I3,1H*,F8.4,F7.2)	RAYN	140
522	ALFA=ALFA*0.01745329	RAYN	141
	IF (MOE .EQ. 0) GO TO 161	RAYN	142
	DEP=DEP/0.3048 & G=G/0.3048 & U=U/0.3048	RAYN	143
	V=V/0.3048 & HGT=HGT/0.3048	RAYN	144
161	KMAX=MAXQ & PX=X & PY=Y & KNUMT=NUMT	RAYN	145
	CALL STORE(X,Y,A,KMAX,TIMEQ,KCIN,KREST)	RAYN	146
	GO TO (10,11) PIT	RAYN	147
11	IF (NPT .EQ. 0) GO TO 10	RAYN	148
	IF (MOD(FUD,LI) .NE. 0) GO TO 3053	RAYN	149
	WRITE (6,7) (PROJECT,DATE1,DATE2,NPLOT,TT,N,DELTAT,CF,KRTOL)	RAYN	150
	WRITE (6,150)	RAYN	151
3053	FUD=FUD+1.	RAYN	152
	WRITE (6,9)	RAYN	153
9	FORMAT (80X,25HPACKET CURVATURE AVERAGED)	RAYN	154
10	IF (MAXQ .GT. 1) GO TO 13	RAYN	155
	GO TO (3,402),NDP	RAYN	156
13	IF (NGO .EQ. 1) GO TO 3	RAYN	157
	WRITE (6,407)	RAYN	158
407	FORMAT (80X,25HRAV REACHED GRID BOUNDARY)	RAYN	159
15	IF (NPT .NE. 0) GO TO 190	RAYN	160
	IF (MOE .EQ. 0) GO TO 212	RAYN	161
	PDEP=PDEP*0.3048 & PG=PG*0.3048 & PU=PU*0.3048	RAYN	162
	PV=PV*0.3048 & PHGT=PHGT*0.3048	RAYN	163
C	WRITE RAY PARTICULARS FOR THE TERMINAL POINT	RAYN	164
212	WRITE (6,854) (N,TT,KMAX,PX,PY,PDEP,ANGLE,GAM,PHGT)	RAYN	165
854	FORMAT (1H*,I3,F6.1,1X,I5,2F7.2,F8.2,2F7.2,F8.4,/))	RAYN	166
190	IF (MAXQ .LE. 1 .OR. NPT .EQ. 0 .OR.	RAYN	167
	8MOD(MAXQ,SK) .EQ. 0) GO TO 1900	RAYN	168
C	RAY PARTICULARS HAVE NOT BEEN WRITTEN FOR THE LAST POINT	RAYN	169
	IF (MOD(FUD,LI) .NE. 0) GO TO 3031	RAYN	170
	WRITE (6,7) (PROJECT,DATE1,DATE2,NPLOT,TT,N,DELTAT,CF,KRTOL)	RAYN	171
	WRITE (6,150)	RAYN	172
3031	IF (MOE .EQ. 0) GO TO 3030	RAYN	173

PDEP=PDEP*0.3048 \$ PG=PG*0.3048 \$ PU=PU*0.3048	RAYN	174
PV=PV*0.3048 \$ PHGT=PHGT*0.3048	RAYN	175
3030 IF (RFLBUM .NE. 0) GO TO 558	RAYN	176
IF (RFRBUM .NE. 0) GO TO 560	RAYN	177
WRITE (6,612) (KMAX,PX,PY,PDEP,ANGLE,GAM,PD,PFK,PALFA,PG,	RAYN	178
\$PU,PV,PHGT,PKS,PKF,PKR,PCTDIF)	RAYN	179
GO TO 1900	RAYN	180
558 RFLBUM=RFRBUM=0	RAYN	181
WRITE (6,613) (KMAX,PX,PY,PDEP,ANGLE,GAM,PD,PFK,PALFA,PG,	RAYN	182
\$PU,PV,PHGT,PKS,PKF,KNUMT,PKR,PCTDIF)	RAYN	183
GO TO 1900	RAYN	184
560 RFRBUM=0.	RAYN	185
WRITE (6,614) (KMAX,PX,PY,PDEP,ANGLE,GAM,PD,PFK,PALFA,PG,	RAYN	186
\$PU,PV,PHGT,PKS,PKF,KNUMT,PKR,PCTDIF)	RAYN	187
1900 CALL DRAW(N,KMAX,KCIN,KREST)	RAYN	188
RETURN	RAYN	189
END	RAYN	190

'SUBROUTINE MOVE(X,Y,A,FK,NGO,MIT,NFK,NOP,AV,LI,NPT)	MOVE	1
DIMENSION S(6,6),EM(6,12),C(12),YVM(6),E(6)	MOVE	2
\$,CHAT(100,100),AX(200),AY(200),CONTUR(9)	MOVE	3
REAL KP,KF,KS,KPTOL,KFC	MOVE	4
INTEGER REF,ROP,REFLOT,REFROCT,FLAG1,FLAG2,FLAG3,RCOUNT,FUD,BRK	MOVE	5
COMMON S,EM,E,YVM,CHAT,C,AX,AY,CONTUR,PROJCT,GRID,CCON,FAN,CATE1	MOVE	6
\$,DATE2,CIN,DIR,ROP,TT,WBCOP,HUE,DY,DELTAT,SLTAT,C,HGT,HGTZ,SVX	MOVE	7
\$,SVY,SDEP,H,DEP,HL,V,SAVV,PFEV,SPREV,U,SAVU,GZER0,G,SG,SVG,CUD,KS	MOVE	8
\$,DGOX,SAV,TPI,SAV,SAVV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHOK	MOVE	9
\$,SVFKB,SAVFK,FKEAR,FAVQ,SK,FUD,NUMT,INUY,IFLG,FCOUNT,AM,ANN	MOVE	10
\$,REFLOT,RFLBUM,REFROCT,RFRBUM,BRK,FLAG1,FLAG2,FLAG3,FLAGF,KFC,CF,BZ	MOVE	11
\$,SBZ,BDZ,SBZ,KPTOL,KP,POT,F1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5	MOVE	12
C IN THIS SUBROUTINE THE PATH OF THE WAVE PACKET IS DETERMINED.	MOVE	13
C TESTS ARE MADE TO LOCATE A REFLECTION POINT, AND IF DESIRED	MOVE	14
C THE RAY PATH IS CONTINUED BEYOND THE REFLECTION POINT.	MOVE	15
C SUBROUTINES SURFACE AND HEIGHT ARE CALLED.	MOVE	16
NUMT=1 \$ MIT=1	MOVE	17
C SAVE VALUES IN CASE OF BREAK UP OF TIME STEP INTERVAL	MOVE	18
SVG=G	MOVE	19
IF (MAXQ .NE. 2) GO TO 3033	MOVE	20
SVFKB=FK	MOVE	21
GO TO 202	MOVE	22
3033 SVFKB=FKEAR \$ SAVFK=FK	MOVE	23
C COMPUTE THE INCREMENTAL DISTANCE TO THE NEXT RAY POINT	MOVE	24
202 D=(G*SOLTAT)/GRID	MOVE	25
203 IF (MAXC-2) 38,102,104	MOVE	26
102 FKEAR=FK	MOVE	27
C CHECK FOR TIME STEP BREAK UP DUE TO BETA CALCULATION OR REFLECTION	MOVE	28
104 IF (REFROCT .NE. 0 .OR. REFLOT .NE. 0	MOVE	29
\$.OR. ABS(TAN(SAV-SVALFA)) .LE. 3.7320508	MOVE	30
\$.OR. ABS(TAN(A-SVALFA)) .GE. 3.7320508) GO TO 81	MOVE	31
REFLOT=1 \$ RFLBUM=1	MOVE	32
C ITERATE TO FIND VALUES FOR THE NEXT POINT	MOVE	33
81 DO 20 IT=1,50	MOVE	34
39 DELA=FKEAR*D \$ AA=A+DELA \$ ABAR=A+0.5*DELA	MOVE	35
DELX=D*COS(ABAR) \$ DELY=D*SIN(ABAR) \$ XX=X+DELX \$ YY=Y+DELY	MOVE	36
CALL SURFACE(XX,YY,AA,FKK,NFK,NOP,AAV)	MOVE	37
AVP=AAV-ALFA	MOVE	38
IF (FLAG2 .EQ. 0) GO TO 86	MOVE	39

REF=1	MOVE	40
GO TO 13	MOVE	41
86 DUC=SAVV/PREV	MOVE	42
GO TO (101,6,38,38,38,38,38,38),MIT	MOVE	43
101 IF (NDF .EQ. 2) GO TO 33	MOVE	44
FKBAR=0.5*(FK+FKK)	MOVE	45
IF (IT .NE. 49) GO TO 88	MOVE	46
SVFK=FKBAR	MOVE	47
88 IF (IT-48) 5,37,9	MOVE	48
37 FKKPP=FKBAR	MOVE	49
5 IF (MAXQ .GT. 2) GO TO 9	MOVE	50
IF (IT .LE. 1) GO TO 20	MOVE	51
C TEST THE CONVERGENCE OF THE RAY CURVATURE CALCULATIONS	MOVE	52
9 IF (ABS(FKKPP-FKBAR) .LE. 0.00009/D) GO TO 6	MOVE	53
20 FKKPP=FKBAR	MOVE	54
IF (ABS(FKKPP-FKBAR) .LE. 0.00009/D) GO TO 18	MOVE	55
C DETERMINE IF CONVERGENCE FAILED DUE TO A REFLECTION POINT	MOVE	56
IF (DUD .GT. 1.0 .AND.	MOVE	57
\$ABS(TAN(AVP)) .GT. 5.6712818) GO TO 91	MOVE	58
MIT=3	MOVE	59
GO TO 38	MOVE	60
91 REF=2	MOVE	61
GO TO 13	MOVE	62
18 FKBAR=.5*(FKBAR+SVFK) & MIT=2	MOVE	63
GO TO 39	MOVE	64
C DETERMINE IF TOO CLOSE TO A REFLECTION POINT	MOVE	65
6 IF (DUD .LE. 1.0 .OR.	MOVE	66
\$ABS(TAN(AVP)) .LE. 114.588650 .OR.	MOVE	67
\$ABS(TAN(A-SVALFA)) .GE. 3.7320518) GO TO 92	MOVE	68
REF=3	MOVE	69
C BEGIN REFLECTION	MOVE	70
13 IF (NPT .EQ. 0) GO TO 14	MOVE	71
IF (MOD(FUD,LI) .NE. 0) GO TO 3043	MOVE	72
C WRITE PAGE AND COLUMN HEADINGS	MOVE	73
WRITE (6,96) (PROJECT,DATE1,DATE2,NPLOT,TT,N,SOLTAT,CF,KRTOL)	MOVE	74
96 FORMAT (1H1,11HPROJECT NO.,A6,1H,,2X,2A6,1H,,5X,8HPLOT NO.,I3,1H,,	MOVE	75
1X,7HPERIOD=,F5.1,4HSEC.,1H,,1X,7HRAY NO.,I3,1H,,1X,7HDELTA=,	MOVE	76
\$F6.2,1H,,1X,3HCF=,F3.0,1H,,1X,6HKRTOL=,F8.6,/))	MOVE	77
WRITE (6,150)	MOVE	78
150 FORMAT (1X,3HMAX,1X,1HX,6X,1HY,6X,1HH,7X,4HPACK,3X,4HHAWE,3X,1HD,	MOVE	79
\$9X,2HFK,8X,4HALFA,5X,1HG,5X,1HU,5X,	MOVE	80
\$1HV,5X,3HHGT,5X,2HKS,6X,2HKF,7X,2HNO,1X,2HNR,7X,6HPCTDIF,/))	MOVE	81
3043 PACK=A*57.29577951 & PACK=CNVPSA-PACK+180.	MOVE	82
HAWE=AV*57.29577951 & HAWE=CNVPSA-HAWE+180. & KMAX=MAXQ-1	MOVE	83
SVALFA=SVALFA*57.29577951	MOVE	84
IF (MOE .EQ. 0) GO TO 210	MOVE	85
SDEP=SDEP*0.3048 & SG=SG*0.3048 & SAVU=SAVU*0.3048	MOVE	86
SAVV=SAVV*0.3048 & HGT=HGT*0.3048	MOVE	87
C WRITE RAY PARTICULARS	MOVE	88
210 WRITE (6,151) (KMAX,X,Y,SDEP,PACK,HAWE,SD,FK,SVALFA,SG,	MOVE	89
\$SAVU,SAVV,HGT,KS,KFC,KR)	MOVE	90
151 FORMAT (1X,1H*,I3,2F7.2,F8.2,2F7.2,2E10.2,F8.2,1X,3F6.2,3F8.4	MOVE	91
\$,4X,F9.4)	MOVE	92
FUD=FUD+1 & SVALFA=SVALFA*0.01745329	MOVE	93
IF (MOE .EQ. 0) GO TO 212	MOVE	94
SDEP=SDEP/0.3048 & SG=SG/0.3048 & SAVU=SAVU/0.3048	MOVE	95
SAVV=SAVV/0.3048 & HGT=HGT/0.3048	MOVE	96
212 IF (MOD(FUD,LI) .NE. 0) GO TO 3044	MOVE	97
WRITE (6,96) (PROJECT,DATE1,DATE2,NPLOT,TT,N,SOLTAT,CF,KRTOL)	MOVE	98
WRITE (6,150)	MOVE	99

C WRITE TYPE OF REFLECTION	MOVE	100
3044 GO TO (97,98,99),REF	MOVE	101
97 WRITE (6,152)	MOVE	102
152 FORMAT (1X,43HREFLECTION: SNELLS LAW WITH PHASE VELOCITY)	MOVE	103
GO TO 300	MOVE	104
98 WRITE (6,153)	MOVE	105
153 FORMAT (1X,44HREFLECTION: PACKET CURVATURE ITERATION NOT	MOVE	106
3,10HCONVERGING)	MOVE	107
GO TO 300	MOVE	108
99 WRITE (6,154)	MOVE	109
154 FORMAT (1X,34HREFLECTION: NEAR REFLECTION POINT)	MOVE	110
300 FUD=FUD+1.	MOVE	111
14 IF (POP.NE. 0) GO TO 301	MOVE	112
MIT=6	MOVE	113
GO TO 38	MOVE	114
301 FLAG2=0.	MOVE	115
C COMPUTE REFLECTION ANGLES	MOVE	116
SAV=2.*SVALFA-SAV+3.1415927 \$ A=2.*SVALFA-A+3.1415927 \$ AV=SAV	MOVE	117
RCOUNT=RCOUNT+1.	MOVE	118
C TEST FOR REFLECTION HANG-UP	MOVE	119
IF (FCOUNT.LT. 2) GO TO 305	MOVE	120
MIT=7	MOVE	121
GO TO 38	MOVE	122
305 FLAG1=1.	MOVE	123
CALL SURFCE(X,Y,A,FK,NFK,NOP,SAV)	MOVE	124
FLAG1=0. \$ PATI=POT \$ QATI=QOT \$ FLAGR=1.	MOVE	125
GO TO 102	MOVE	126
C DETERMINE IF POINT IS TOO CLOSE TO A GRID BOUNDARY	MOVE	127
92 IF ((XX-1.5)*((AMM-1.5)-XX) .GE. 0.0 .AND.	MOVE	128
\$(YY-1.5)*((ANN-1.5)-YY) .GE. 0.0) GO TO 309	MOVE	129
NGO=2	MOVE	130
309 SVX=X \$ SVY=Y \$ XS=XX \$ YS=YY \$ SSALFA=SVALFA \$ SVALFA=ALFA	MOVE	131
SVA=A \$ SAV=SAV \$ SAV=AAV \$ FCOUNT=0.	MOVE	132
SPREV=PREV \$ PREV=SAVV \$ SDEP=DEP \$ SAVV=V \$ SAVU=U \$ SG=G	MOVE	133
AAA=.5*(AA+A) \$ A=AA \$ AAV=.5*(AAV+AV) \$ AV=AAV \$ FK=FKK \$ SJ=0	MOVE	134
IF (REFLOT.EQ. 1) GO TO 40	MOVE	135
C COMPUTE P AND Q FOR THE INTERMEDIATE POINTS	MOVE	136
XX=X+(1./3.)*DELX*(ABS(COS(AAA)))	MOVE	137
YY=Y+(1./3.)*DELY*(ABS(SIN(AAA))) \$ FLAG1=1.	MOVE	138
CALL SURFCE(XX,YY,AAA,FKK,NFK,NOP,AAAV)	MOVE	139
P1=POT \$ Q1=QOT \$ XX=X+.4*DELX*(ABS(COS(AAA)))	MOVE	140
YY=Y+.4*DELY*(ABS(SIN(AAA)))	MOVE	141
CALL SURFCE(XX,YY,AAA,FKK,NFK,NOP,AAAV)	MOVE	142
P2=POT \$ Q2=QOT \$ XX=X+.45573725*DELX*(ABS(COS(AAA)))	MOVE	143
YY=Y+.45573725*DELY*(ABS(SIN(AAA)))	MOVE	144
CALL SURFCE(XX,YY,AAA,FKK,NFK,NOP,AAAV)	MOVE	145
P3=POT \$ Q3=QOT \$ XX=X+(2./3.)*DELX*(ABS(COS(AAA)))	MOVE	146
YY=Y+(2./3.)*DELY*(ABS(SIN(AAA)))	MOVE	147
CALL SURFCE(XX,YY,AAA,FKK,NFK,NOP,AAAV)	MOVE	148
P4=POT \$ Q4=QOT \$ XX=X+.8*DELX*(ABS(COS(AAA)))	MOVE	149
YY=Y+.8*DELY*(ABS(SIN(AAA)))	MOVE	150
CALL SURFCE(XX,YY,AAA,FKK,NFK,NOP,AAAV)	MOVE	151
P5=POT \$ Q5=QOT	MOVE	152
CALL SURFCE(XS,YS,AA,FKK,NFK,NOP,AAV)	MOVE	153
FLAG1=0	MOVE	154
40 X=XS \$ Y=YS	MOVE	155

50

CALL HEIGHT(X,Y,A,FK,NGO,MIT,NFK,NDF,AV)	MOVE	156
IF (BPK.EQ. 1) GO TO 203	MOVE	157
C PLACE ANGLES IN THE RANGE J TO 360 DEGREES FOR THE PRINTED OUTPUT.	MOVE	158
CNVPSA=CNVRSA*.0174532925 & A=CNVRSA-A+3.1415927	MOVE	159
51 IF (A.GE. 0.0) GO TO 50	MOVE	160
A=A+6.2831853	MOVE	161
GO TO 51	MOVE	162
50 IF (A.LT. 6.2831853) GO TO 52	MOVE	163
A=A-6.2831853	MOVE	164
GO TO 50	MOVE	165
52 A=CNVRSA-A+3.1415927	MOVE	166
AV=CNVRSA-AV+3.1415927	MOVE	167
54 IF (AV.GE. 0.0) GO TO 53	MOVE	168
AV=AV+6.2831853	MOVE	169
GO TO 54	MOVE	170
53 IF (AV.LT. 6.2831853) GO TO 55	MOVE	171
AV=AV-6.2831853	MOVE	172
GO TO 53	MOVE	173
55 AV=CNVRSA-AV+3.1415927 & CNVRSA=CNVRSA*57.29577951	MOVE	174
38 RETURN	MOVE	175
END	MOVE	176

SUBROUTINE HEIGHT(X,Y,A,FK,NGO,MIT,NFK,NDF,AV)	HEIGHT	1
DIMENSION S(6,6),EM(6,12),C(12),YVM(6),E(6)	HEIGHT	2
&,CHAT(100,100),AX(2000),AY(2000),CONTR(9)	HEIGHT	3
REAL KP,KF,KS,KRTOL,KFC	HEIGHT	4
&,L1,L2,L3,L4,L5,L6,L7,L8,L9,K1,K2,K3,K4,K5,K6,K7,K8,K9	HEIGHT	5
INTEGER DX1,DX2,WBCOP,REFLOT,REFROT,FLAGR,REFLUM,REFBUM,BRK	HEIGHT	6
COMMON S,EM,E,YVM,CHAT,C,AX,AY,CONTR,PROJECT,GRID,CON,FAN,DATE1	HEIGHT	7
&,DATE2,CIN,DIR,ROP,TT,WBCOP,MOE,DY,DELTAT,SOLTAT,D,HGT,HGTZ,SVX	HEIGHT	8
&,SVY,SDEP,W,DEP,WL,V,SAVV,PFEV,SPREV,U,SAVU,GZERO,G,SG,SVG,GUD,KS	HEIGHT	9
&,DGDY,SAV,TPI,SAV,SAVV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX	HEIGHT	10
&,SVFKB,SAVFK,FKPAR,MAXQ,SK,FUD,NUIT,INUM,IFLG,RCOUNT,ANN,ANN	HEIGHT	11
&,REFLOT,REFBUM,REFROT,REFBUM,BRK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,DF,SBZ	HEIGHT	12
&,SBZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5	HEIGHT	13
C IN THIS SUBROUTINE THE WAVE HEIGHT IS COMPUTED. THE TIME STEP	HEIGHT	14
C IS SUCCESSIVELY HALVED, IF NECESSARY, TO MAINTAIN THE DESIRED	HEIGHT	15
C ACCURACY IN COMPUTING THE REFRACTION COEFFICIENT, OR THE RAY	HEIGHT	16
C PATH IF NEAR A REFLECTION POINT.	HEIGHT	17
IF (MAXQ.GT. 1) GO TO 2	HEIGHT	18
PATI=POT & CATI=QOT & BZTOL=KRTOL**2	HEIGHT	19
C COMPUTE INITIAL D(BETA)/DT	HEIGHT	20
BDZ=-TAN(A-ALFA)*SIN(A-ALFA)*CGDX/GRID	HEIGHT	21
KF=BZ=KR=KS=1. & HGT=HGTZ*KS*KF*KR & KFC=KF	HEIGHT	22
GO TO 38	HEIGHT	23
C COMPUTE SHOALING COEFFICIENT	HEIGHT	24
2 KS=SQRT(ABS(GZERO/G))	HEIGHT	25
C COMPUTE FRICTION COEFFICIENT	HEIGHT	26
SKH=6.283185308*DEP/(V*TT)	HEIGHT	27
KF=1./(KFC*.8195*DF*HGTZ*D*GRID/((TT**3)*GZERO)*	HEIGHT	28
&(2.*KS/(EXP(SKH)-EXP(-SKH)))**3+1.)	HEIGHT	29
KFC=KFC*KF & PSAV=PATI & SBZ=PZ & SBDZ=BDZ	HEIGHT	30
IF (NFK.EQ. 1) GO TO 30	HEIGHT	31
IF (REFLOT.EQ. 0) GO TO 33	HEIGHT	32
C COMPUTE BETA AND D(BETA)/DT ANALYTICALLY NEAR A REFLECTION POINT	HEIGHT	33
PREA=A-DELA-ALFA & TPII=COS(PREA)/BZ & TPI=ACOS(TPII)	HEIGHT	34
BZ=COS(A-ALFA)/COS(TPI)	HEIGHT	35
BDZ=-(SIN(A-ALFA)**2)/COS(TPI)*CGDX/GRID	HEIGHT	36
IF (FLAGR.EQ. 0) GO TO 71	HEIGHT	37
ROP=0	HEIGHT	38
GO TO 71	HEIGHT	39
33 IF (ABS(DHDX/GRID).GT. 0.00001) GO TO 31	HEIGHT	40
30 BDZ=EBZ=EBDZ=0.	HEIGHT	41
GO TO 71	HEIGHT	42

31	IF (FLAGR .EQ. 0) GO TO 32	HEIGHT	43
	ROP = 0	HEIGHT	44
	C COMPUTE BETA AND D(BETA)/DT USING RUNGE KUTTA METHOD	HEIGHT	45
32	K1=DELTA T*BDZ \$ L1=-DELTA T*(PATI*BDZ+QATI*BZ)	HEIGHT	46
	K2=DELTA T*(BDZ+.4*L1)	HEIGHT	47
	L2=-DELTA T*(P2*(BDZ+.4*L1)+Q2*(BZ+.4*K1))	HEIGHT	48
	K3=DELTA T*(BDZ+.29697761*L1+.0.15875964*L2)	HEIGHT	49
	L3=-DELTA T*(P3*(BDZ+.29697761*L1+.0.15875964*L2)+Q3*(BZ+	HEIGHT	50
	\$.29697761*K1+.0.15875964*K2))	HEIGHT	51
	K4=DELTA T*(BDZ+.21810040*L1-3.05096516*L2+3.83286476*L3)	HEIGHT	52
	L4=-DELTA T*(POT*(BDZ+.21810040*L1-3.05096516*L2+3.83286476*L3)	HEIGHT	53
	\$.QOT*(BZ+.21810040*K1-3.05096516*K2+3.83286476*K3))	HEIGHT	54
	K5=DELTA T*(BDZ+L1/3.)	HEIGHT	55
	L5=-DELTA T*(P1*(BDZ+L1/3.)+Q1*(BZ+K1/3.))	HEIGHT	56
	K6=DELTA T*(BDZ+(6.*L5+.4.*L1)/25.)	HEIGHT	57
	L6=-DELTA T*(P2*(BDZ+(6.*L5+.4.*L1)/25.)+Q2*(BZ+(6.*K5+.4.*K1)/25.))	HEIGHT	58
	K7=DELTA T*(BDZ+(15.*L6-12.*L5+L1)/4.)	HEIGHT	59
	L7=-DELTA T*(POT*(BDZ+(15.*L6-12.*L5+L1)/4.)+QOT*(BZ+(15.*K6	HEIGHT	60
	\$.12.*K5+K1)/4.))	HEIGHT	61
	K8=DELTA T*(BDZ+(8.*L7-50.*L6+90.*L5+6.*L1)/81.)	HEIGHT	62
	L8=-DELTA T*(P4*(BDZ+(8.*L7-50.*L6+90.*L5+6.*L1)/81.)+Q4*(BZ+	HEIGHT	63
	\$(8.*K7-50.*K6+90.*K5+6.*K1)/81.))	HEIGHT	64
	K9=DELTA T*(BDZ+(8.*L7+10.*L6+36.*L5+6.*L1)/75.)	HEIGHT	65
	L9=-DELTA T*(P5*(BDZ+(8.*L7+10.*L6+36.*L5+6.*L1)/75.)+Q5*(BZ+	HEIGHT	66
	\$(8.*K7+10.*K6+36.*K5+6.*K1)/75.))	HEIGHT	67
	BZ5=BZ+(1./192.)*(23.*K1+125.*K6-81.*K8+125.*K9)	HEIGHT	68
	BDZ5=BDZ+(1./192.)*(23.*L1+125.*L6-81.*L8+125.*L9)	HEIGHT	69
	BZ=BDZ+.17476028*L1-.0.55148066*L2+.1.20553560*L3+.0.17118478*L4	HEIGHT	70
	BZ=BDZ+.17476028*K1-.0.55148066*K2+.1.20553560*K3+.0.17118478*K4	HEIGHT	71
	C COMPUTE DIFFERENCE BETWEEN 4TH AND 5TH ORDER SOLUTIONS	HEIGHT	72
	EBZ=BZ-BZ5 \$ EBDZ=BDZ-BDZ5	HEIGHT	73
	C COMPUTE REFRACTION COEFFICIENT	HEIGHT	74
71	KR=1./(SQRT(ABS(BZ)))	HEIGHT	75
	IF (REFLECT .EQ. 0) GO TO 401	HEIGHT	76
	IF (IFLG .NE. 0) GO TO 55	HEIGHT	77
	C NEAR A REFLECTION POINT LIMIT THE CHANGE IN THE PACKET DIRECTION	HEIGHT	78
	IF (ABS(DELTA) .LT. 0.017453293) GO TO 22	HEIGHT	79
	GO TO 58	HEIGHT	80
401	IF (IFLG .NE. 0) GO TO 55	HEIGHT	81
	C REQUIRE THAT THE BETA CALCULATION HAS THE DESIRED ACCURACY	HEIGHT	82
	IF (ABS(EBZ) .GE. BZTOL .OR. ABS(EBDZ) .GE. BZTOL) GO TO 21	HEIGHT	83
	22 IF (NUMT .LE. 1) GO TO *	HEIGHT	84
	IFLG=1	HEIGHT	85
	GO TO 55	HEIGHT	86
21	REFRCT=1. \$ RFRBUN=1.	HEIGHT	87
	C BREAK UP TIME STEP INTERVAL AND RESUME CALCULATIONS	HEIGHT	88
58	DELTA T=.5*DELTA T	HEIGHT	89
	IF (DELTA T .GE. 0.5) GO TO 81	HEIGHT	90
	MIT=8 \$ BRK=0.	HEIGHT	91
	GO TO 38	HEIGHT	92
81	G=SVG \$ D=G*DELTA T/GRID \$ BRK=1. \$ NUMT=2*NUMT	HEIGHT	93
	C RECOVER SAVED VALUES	HEIGHT	94
		HEIGHT	95
		HEIGHT	96

¹This statement has been removed.

IF (MAXQ .NE. 2) GO TO 59	HEIGHT	97
FK=SVFKB	HEIGHT	98
GO TO 61	HEIGHT	99
59 FKBAR=SVFKB \$ FK=SAVFK	HEIGHT	100
61 A=SVA \$ SAV=SAV \$ SAVV=PREV \$ PREV=SPREV \$ SVALFA=SSALFA	HEIGHT	101
X=SVX \$ Y=SVY \$ BZ=SBZ \$ BDZ=SDZ \$ PATI=PSAV \$ KFC=KFC/KF	HEIGHT	102
GO TO 38	HEIGHT	103
55 INUM=INUM+1.	HEIGHT	104
IF (INUM .LT. NUMT) GO TO 64	HEIGHT	105
C RESUME CALCULATIONS WITH ORIGINAL TIME STEP	HEIGHT	106
IFLG=0 \$ INUM=0 \$ DELTAT=SOLTAT \$ BRK=0	HEIGHT	107
D=G*DELTAT/GRID	HEIGHT	108
GO TO 4	HEIGHT	109
64 D=G*DELTAT/GRID \$ PATI=POT \$ QATI=QOT	HEIGHT	110
C TEST FOR FOCAL POINT OR CAUSTIC	HEIGHT	111
IF (BZ .GT. 0.) GO TO 67	HEIGHT	112
68 MIT=4 \$ BRK=0.	HEIGHT	113
GO TO 38	HEIGHT	114
67 BRK=1.	HEIGHT	115
GO TO 38	HEIGHT	116
4 IF (BZ .LE. 0.) GO TO 68	HEIGHT	117
PATI=POT \$ QATI=QOT \$ REFRCT=0 \$ REFLOT=0	HEIGHT	118
C COMPUTE WAVE HEIGHT	HEIGHT	119
HGT=HGTZ*KS*KFC*KR	HEIGHT	120
IF (HBCOP .EQ. 0) GO TO 38	HEIGHT	121
C TEST FOR WAVE BREAK	HEIGHT	122
IF (HGT/(V*IT) .LE. (1./7.)*TANH(SKH)) GO TO 38	HEIGHT	123
MIT=5	HEIGHT	124
38 RETURN	HEIGHT	125
END	HEIGHT	126
SUBROUTINE SURFCE(X,Y,A,FK,NFK,NDP,AV)	SURFACE	1
DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)	SURFACE	2
\$,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)	SURFACE	3
REAL KF,KF,KS,KKTOL,KFC	SURFACE	4
INTEGER FLAG1,FLAG2	SURFACE	5
COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1	SURFACE	6
\$,DATE2,CIN,DIP,ROP,IT,HBCOP,MOE,DY,DELTAT,SOLTAT,D,HGT,HGTZ,SVX	SURFACE	7
\$,SVY,SDEP,W,DEP,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,BUD,KS	SURFACE	8
\$,DGOX,SVA,TPI,SAV,SAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHGX	SURFACE	9
\$,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,PCOUNT,AMP,ANN	SURFACE	10
\$,REFLOT,REFLUM,REFRCT,REFRUM,ERK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZ	SURFACE	11
\$,SBZ,BDZ,SDZ,KKTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5	SURFACE	12
C IN THIS SUBROUTINE THE WATER DEPTH, ROTATION ANGLE, WAVELET	SURFACE	13
C DIRECTION, GEOMETRIC GROUP VELOCITY, COEFFICIENTS OF THE RAY	SURFACE	14
C SEPARATION EQUATION, AND THE PACKET RAY CURVATURE ARE COMPUTED.	SURFACE	15
C SUBROUTINES VELCTY AND CONDER ARE CALLED.	SURFACE	16
I=X \$ J=Y \$ FI=I \$ FJ=J \$ XL=X+1.-FI \$ YL=Y+1.-FJ	SURFACE	17
IF (MAXQ .LE. 1) GO TO 1	SURFACE	18
IF (ZI .NE. FI) GO TO 1	SURFACE	19
IF (ZJ .EQ. FJ) GO TO 3	SURFACE	20
1 ZI=FI \$ ZJ=FJ	SURFACE	21

C SELECT 12 WATER DEPTHS ABOUT RAY POINT	SURFACE	22
C(1)=CMAT(J+1,I) & C(2)=CMAT(J+2,I) & C(3)=CMAT(J,I+1)	SURFACE	23
C(4)=CMAT(J+1,I+1) & C(5)=CMAT(J+2,I+1) & C(6)=CMAT(J+3,I+1)	SURFACE	24
C(7)=CMAT(J,I+2) & C(8)=CMAT(J+1,I+2) & C(9)=CMAT(J+2,I+2)	SURFACE	25
C(10)=CMAT(J+3,I+2) & C(11)=CMAT(J+1,I+3) & C(12)=CMAT(J+2,I+3)	SURFACE	26
C FIT QUADRATIC SURFACE TO 12 WATER DEPTHS	SURFACE	27
DO 318 II=1,6	SURFACE	28
YVW(II)=0.	SURFACE	29
DO 318 L=1,12	SURFACE	30
YVW(II)=YVW(II)+C(L)*E4(II,L)	SURFACE	31
318 CONTINUE	SURFACE	32
DO 319 II=1,6	SURFACE	33
E(II)=0.	SURFACE	34
DO 319 JJ=1,6	SURFACE	35
E(II)=E(II)+S(JJ,II)*YVW(JJ)	SURFACE	36
319 CONTINUE	SURFACE	37
C COMPUTE INTERPOLATED WATER DEPTH	SURFACE	38
3 DEF=(E(1)+E(2)*XL+E(3)*YL+E(4)*XL**2+E(5)*XL*YL+E(6)*YL**2)*DCON	SURFACE	39
C COMPUTE PARTIAL DERIVATIVES OF WATER DEPTH IN FIXED XY-SYSTEM	SURFACE	40
HX=(E(2)+2.*E(4)*XL+E(5)*YL)*DCON	SURFACE	41
HY=(E(3)+E(5)*XL+E(6)*2.*YL)*DCON	SURFACE	42
HXX=2.*E(4)*DCON & HYY=2.*E(6)*DCON & HXY=E(5)*DCON	SURFACE	43
IF (DEF .GT. 0.) GO TO 324	SURFACE	44
NDF=2	SURFACE	45
GO TO 403	SURFACE	46
324 IF (DEF/HL .GT. .64) GO TO 322	SURFACE	47
NFK=2	SURFACE	48
GO TO 323	SURFACE	49
322 NFK=1	SURFACE	50
323 CALL VELCTY(V,TT,MAXQ,DEF,NFK,U)	SURFACE	51
IF (NFK .EQ. 2) GO TO 402	SURFACE	52
W=0.	SURFACE	53
GO TO 10	SURFACE	54
402 DN=1.	SURFACE	55
CALL CONDER(DN,TT,V,MAXQ,NFK)	SURFACE	56
W=DN	SURFACE	57
10 VX=W*HX & VY=W*HY & DHDX=SQRT((HX**2)+(HY**2))	SURFACE	58
IF (ABS(DHDX/GRID) .GT. 0.00001) GO TO 8	SURFACE	59
GO TO 9	SURFACE	60
C COMPUTE ROTATION ANGLE	SURFACE	61
8 ALFA=ATAN2(HY,HX)	SURFACE	62
9 IF (FLAG1 .NE. 0) GO TO 12	SURFACE	63
C COMPUTE WAVELET DIRECTION IN ROTATED XY-SYSTEM USING SNELLS LAW	SURFACE	64
C WITH V AND TEST FOR TOTAL REFLECTION DUE TO THE WAVELETS	SURFACE	65
GP=SAV-ALFA	SURFACE	66
14 IF (ABS(GP) .LE. 6.2831853) GO TO 13	SURFACE	67
IF (GP) 16,13,17	SURFACE	68
16 GP=GP+6.2831853	SURFACE	69
GO TO 14	SURFACE	70
17 GP=GP-6.2831853	SURFACE	71
GO TO 14	SURFACE	72
13 ARG1=V*SIN(GP)/SAVV	SURFACE	73
IF (ABS(ARG1) .LE. 1.) GO TO 18	SURFACE	74
FLAG2=1.	SURFACE	75
GO TO 403	SURFACE	76
18 FLAG2=0. & GPT=ASIN(ARG1)	SURFACE	77

IF (ABS(GP) .LE. 4.7123889) GO TO 20	SURFACE	78
AVP=6.2831853+GPT	SURFACE	79
GO TO 22	SURFACE	80
20 IF (ABS(GP) .LE. 1.5707963) GO TO 23	SURFACE	81
AVP=3.1415927-GPT	SURFACE	82
GO TO 22	SURFACE	83
23 AVP=GPT	SURFACE	84
22 AV=AVP*ALFA	SURFACE	85
12 PHI=A-AV \$ G=U*COS(PHI)	SURFACE	86
DVDX=N*DHDX \$ BAR3=12.5663708/TT \$ BAR4=BAR3*DEP/V	SURFACE	87
IF (NFK .EQ. 2) GO TO 25	SURFACE	88
DUCX=DVDX/2.	SURFACE	89
GO TO 27	SURFACE	90
25 DUDX=(1./(EXP(BAR4)-EXP(-BAR4)))*(BAR3*DHDX-BAR4*(BAR3*DHDX-BAR4	SURFACE	91
\$*DVDX)*((EXP(BAR4)+EXP(-BAR4))/(EXP(BAR4)-EXP(-BAR4)))+DVDX*(EXP(SURFACE	92
\$BAR4)-EXP(-BAR4))/2.)	SURFACE	93
27 RHO=1./(1.+TAN(PHI)*TAN(A-ALFA)) \$ SIGMA=U*SIN(PHI)*TAN(AV-ALFA)/V	SURFACE	94
DGDXX=RHO*(DUDX*COS(PHI)+SIGMA*DVDX)	SURFACE	95
IF (NFK .EQ. 2) GO TO 29	SURFACE	96
POT=0. \$ QOT=0. \$ FK=J.	SURFACE	97
GO TO 403	SURFACE	98
C COMPUTE P IN ROTATED XY-SYSTEM	SURFACE	99
28 POT=(-2.*COS(A-ALFA)*JGX)/GRID \$ DAVDX=TAN(AV-ALFA)*DVDX/V	SURFACE	100
DAUX=TAN(A-ALFA)*DGDXX/G \$ DPHIDX=DAUX-DAVDX	SURFACE	101
DRHODX=-(RHO**2)*(TAN(A-ALFA)*DPHIDX/(COS(PHI)**2)+	SURFACE	102
\$TAN(PHI)*DAUX/(COS(A-ALFA)**2))	SURFACE	103
DSIGDX=SIGMA*(DUDX/U-DVDX/V)+U*(COS(PHI)*TAN(AV-ALFA)*DPHIDX+	SURFACE	104
\$SIN(PHI)*DAVDX/(COS(AV-ALFA)**2))/V	SURFACE	105
DHODX=(COS(ALFA)**2)*HXX+2.*SIN(ALFA)*COS(ALFA)*HXY+(SIN(ALFA)**	SURFACE	106
\$2)*HYY	SURFACE	107
SMA=6.2831854/(32.2*TT) \$ SMAB=1./64.4	SURFACE	108
DVDXX=N*(DHODX+(DHDX**2)*(-4.*SMAB*(V**2)/((2.*SMAB*(V**2)+DEP*	SURFACE	109
\$ (1.-(SMA*V)**2)**2))	SURFACE	110
DIDX=BAR4*(DHDX/DEP-DVDX/V)	SURFACE	111
DIOXX=(DIOX**2)/BAR4+BAR4*((DHODX-(DHDX**2)/DEP)/DEP-(DVDXX	SURFACE	112
\$-(DVDX**2)/V)/V)	SURFACE	113
30 DUDXX=(1./(EXP(BAR4)-EXP(-BAR4)))*(-(((EXP(BAR4)-EXP(-BAR4))*5+	SURFACE	114
\$BAR4)*DVDX+V*DIOX*(1.-BAR4/TANH(BAR4)))*DIOX/TANH(BAR4)	SURFACE	115
\$+((EXP(BAR4)	SURFACE	116
\$-EXP(-BAR4))*5+BAR4)*DIOXX+DVDX*DIOX*(2.+(EXP(BAR4)+EXP(-BAR4))*	SURFACE	117
\$5-BAR4/TANH(BAR4))+V*(DIOXX*(1.-BAR4/TANH(BAR4))+(DIOX**2)*2/(SURFACE	118
\$EXP(BAR4)-EXP(-BAR4))*(BAR4/((EXP(BAR4)-EXP(-BAR4))*5)-(EXP(BAR4	SURFACE	119
\$)+EXP(-BAR4))*5)))	SURFACE	120
32 DGDXX=RHO*(COS(PHI)*DUDXX+SIGMA*DVDXX)+(COS(PHI)*DRHODX-RHO*SIN	SURFACE	121
\$(PHI)*DPHIDX)*DUDX+(RHO*DSIGDX+SIGMA*DRHODX)*DVDX	SURFACE	122
C COMPUTE Q IN ROTATED XY-SYSTEM	SURFACE	123
QOT=(G*(SIN(A-ALFA)**2)*DGDXX)/(GRID**2)	SURFACE	124
C COMPUTE PACKET RAY CURVATURE IN ROTATED XY-SYSTEM	SURFACE	125
FK=SIN(A-ALFA)*DGDXX/G	SURFACE	126
403 RETURN	SURFACE	127
END	SURFACE	128

SUBROUTINE VELCTY(V,TT,MAXQ,DEP,NFK,U)
C IN THIS SUBROUTINE THE PHASE VELOCITY AND COLLINEAR GROUP
C VELOCITY ARE COMPUTED.

IF (MAXQ .GT. 1) GO TO 102
BAR=6.2831854/TT \$ CXXO=TT*32.2/6.2831854 \$ CCC=CXXO
GO TO 103
102 CCC=XOXY
103 IF (NFK .EQ. 2) GO TO 105
V=CXXO
GO TO 106
105 DO 1000 M=1,90
V=CXXO*TANH(BAR*DEP/CCC)
IF (ABS(V-CCC) .LT. 0.00005) GO TO 106
1000 CCC=(V+CCC)/2.
106 XOXY=V \$ BAR2=2.*BAR*DEP/V
IF (NFK .EQ. 2) GO TO 3036
U=.5*V
GO TO 107
3036 U=.5*V*(1.+2.*BAR2/(EXP(BAR2)-EXP(-BAR2)))
107 RETURN
END

VELCTY	1
VELCTY	2
VELCTY	3
VELCTY	4
VELCTY	5
VELCTY	6
VELCTY	7
VELCTY	8
VELCTY	9
VELCTY	10
VELCTY	11
VELCTY	12
VELCTY	13
VELCTY	14
VELCTY	15
VELCTY	16
VELCTY	17
VELCTY	18
VELCTY	19
VELCTY	20
VELCTY	21

SUBROUTINE CONDER(DN,TT,V,MAXQ,NFK)
C IN THIS SUBROUTINE W=DN IS COMPUTED.
C1=TT/12.5663708 \$ C2=6.2831854/(32.2*TT)
IF (NFK .EQ. 1) GO TO 105
C3=C2*V \$ A1=C3/(1+C3) \$ A2=C3/(1-C3) \$ A3=ALOG(1.+C3)
A4=ALOG(1.-C3) \$ DN=(DN/C1)*(1./((A1+A2+A3+(-A4)))
105 RETURN
END

CONDER	1
CONDER	2
CONDER	3
CONDER	4
CONDER	5
CONDER	6
CONDER	7
CONDER	8

SUBROUTINE PCD(C,E,PCTDIF)
DIMENSION E(6),C(12)
C IN THIS SUBROUTINE THE DIFFERENCE BETWEEN THE WATER DEPTH AND THE
C DEPTH COMPUTED FROM THE 12-POINT SURFACE FIT IS DETERMINED FOR
C THE 4 GRID POINTS CLOSEST TO THE RAY POINT AND THE MAXIMUM
C PERCENTAGE DIFFERENCE OF THE 4 IS DETERMINED.

IF (C(4)*C(5)*C(8)*C(9) .NE. 0.) GO TO 901
PCTDIF=999.
GO TO 902
901 P1=ABS((C(4)-(E(1)+E(2)+E(3)+E(4)+E(5)+E(6)))/C(4))
P2=ABS((C(5)-(E(1)+E(2)+2.*E(3)+E(4)+E(5)+2.*E(6)+E(4)))/C(5))
P3=ABS((C(8)-(E(1)+E(2)+2.*E(3)+E(4)+4.*E(5)+2.*E(6)))/C(8))
P4=ABS((C(9)-(E(1)+E(2)+2.*E(3)+2.*E(4)+4.*E(5)+4.*E(6)+E(4)))/C(9))
PCTDIF=100.*AMAX1(P1,P2,P3,P4)
902 RETURN
END

PCD	1
PCD	2
PCD	3
PCD	4
PCD	5
PCD	6
PCD	7
PCD	8
PCD	9
PCD	10
PCD	11
PCD	12
PCD	13
PCD	14
PCD	15
PCD	16

SUBROUTINE STORF(X,Y,A,KMAX,TIMEQ,KCIN,KREST)
DIMENSION S(6,6),EM(6,12),C(12),YVH(6),E(6)
\$,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)
REAL KP,KF,KS,KRTOL,KFC
COMMON S,EM,E,YVH,CMAT,C,AX,AY,CONTUR,PROJECT,GRID,CCON,FAN,DATE1
\$,DATE2,CIN,DIR,POP,TT,XBCOP,MOE,DY,DELTAT,SOLTAT,D,HGT,HGTZ,SVX
\$,SVY,SDEP,H,DEP,HL,V,SAVV,PFEV,SPREV,U,SAVU,GZERO,G,SG,SVG,CDO,K
\$,DGDY,SAV,TFI,SAV,SAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX
\$,SVFKS,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,PCOUNT,AMM,ANN

STORE	1
STORE	2
STORE	3
STORE	4
STORE	5
STORE	6
STORE	7
STORE	8
STORE	9

\$,REFLECT,RFLBUM,REFRCT,RFRBUM,BFK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZSTORE	10
\$,SBZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5	11
C IN THIS SUBROUTINE THE COORDINATES OF EACH RAY POINT ARE STORED.	12
C IF DESIRED, THE LOCATION OF TICK MARKS AT EQUAL TIME INTERVALS	13
C ALONG THE RAY ARE COMPUTED AND STORED.	14
IF (CIN .LE. 0) GO TO 403	15
IF (KMAX .GT. 1) GO TO 401	16
AT=0.	17
C STORE COORDINATES OF POINT	18
403 KQ=KMAX+KCIN \$ AX(KQ)=X \$ AY(KQ)=Y	19
IF (CIN .LE. 0.) GO TO 205	20
402 ZA=A \$ ZCXY=G	21
GO TO 205	22
401 ET=TIMEQ-AT	23
IF (CIN-ET) 405,404,403	24
C RAY POINT AND TICK MARK COINCIDE, STORE WITH NEGATIVE X	25
404 KQ=KMAX+KCIN \$ AX(KQ)=-X \$ AY(KQ)=Y \$ KREST=KREST+1 \$ AT=AT+CIN	26
GO TO 402	27
C COMPUTE LOCATION OF TICK MARK AND STORE WITH NEGATIVE X	28
405 DSC=(ET-CIN)*(G+ZCXY)*3600./ (GFID*2.)	29
AA=(A+ZA)/2. \$ XM=DSC*COS(AA) \$ YM=DSC*SIN(AA)	30
KQ=KMAX+KCIN \$ AX(KQ)=-X+XM \$ AY(KQ)=Y-YM	31
KREST=KREST+1 \$ KCIN=KCIN+1 \$ AT=AT+CIN	32
GO TO 401	33
205 RETURN	34
END	35

SUBROUTINE DRAW (N,KMAX,KCIN,KREST)	DRAW	1
DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)	DRAW	2
\$,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)	DRAW	3
INTEGER FAN	DRAW	4
REAL KR,KF,KS,KRTOL,KFC	DRAW	5
COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJECT,GFID,DCON,FAN,DATE1	DRAW	6
\$,DATE2,CIN,DIR,ROP,IT,WBCOP,MOE,DY,DELTAT,SLTAT,D,HGT,HGTZ,SVX	DRAW	7
\$,SVY,SDEP,H,DEP,HL,V,SAVV,PPEV,SPPEV,U,SAVU,GZERO,G,SG,SVG,DUJ,KS	DRAW	8
\$,OGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,OHDX	DRAW	9
\$,SVFKB,SAVFK,FKBAR,HAXQ,SK,FUD,NUFT,INUM,IFLG,PCOUNT,AMM,ANN	DRAW	10
\$,REFLECT,RFLBUM,REFRCT,RFRBUM,BFK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZ	DRAW	11
\$,SBZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5	DRAW	12
C IN THIS SUBROUTINE THE RAYS ARE DRAWN AND NUMBERED. IF DESIRED,	DRAW	13
C TICK MARKS ARE DRAWN ON THE RAY AT EQUAL TIME INTERVALS.	DRAW	14
C SUBROUTINES NUMBER AND PLOT ARE CALLED.	DRAW	15
XN=N \$ KMAX=KMAX+KCIN	DRAW	16
IF (AX(KMAX) .GE. 0.) GO TO 601	DRAW	17
AX(KMAX)=-AX(KMAX) \$ KREST=KREST-1	DRAW	18
601 IF (MOD(N,2) .NE. 0) GO TO 104	DRAW	19
C BEGIN EVEN-NUMBERED RAY WITH THE TERMINAL POINT	DRAW	20
KTWO=KMAX-1 \$ KADD=-1 \$ LAST=1 \$ MC=KREST+1	DRAW	21
IF (FAN .EQ. 0) GO TO 201	DRAW	22
CALL NUMBER (AX(KMAX)/DY,AY(KMAX)/DY,3.1,XN,0.0,-1)	DRAW	23
201 CALL PLOT (AX(KMAX)/DY,AY(KMAX)/DY,3)	DRAW	24
IF (KMAX .LE. 1) GO TO 106	DRAW	25
GO TO 105	DRAW	26
C BEGIN ODD-NUMBERED RAY WITH THE INITIAL POINT	DRAW	27

104	KTWO=2 \$ KADD=1 \$ LAST=KMAX \$ MC=0.	DRAW	28
	IF (FAN .NE. 0) GO TO 111	DRAW	29
	C NUMBER RAY AT THE INITIAL POINT	DRAW	30
	CALL NUMBER (AX(1)/DY,AY(1)/DY,0.1,XN,0.0,-1)	DRAW	31
111	CALL PLOT (AX(1)/DY,AY(1)/DY,3)	DRAW	32
	IF (KMAX .LE. 1) GO TO 106	DRAW	33
105	IF (CIN .LE. 0.) GO TO 300	DRAW	34
	IF (AX(KTWO) .LT. 0.) GO TO 302	DRAW	35
	C DRAW SEGMENT OF RAY	DRAW	36
300	CALL PLOT (AX(KTWO)/DY,AY(KTWO)/DY,2)	DRAW	37
	GO TO 303	DRAW	38
302	AX(KTWO)=-AX(KTWO) \$ WI=.05 \$ MC=MC+KADD	DRAW	39
	IF (MOD(MC,10) .NE. 0) GO TO 500	DRAW	40
	WI=.10	DRAW	41
500	XPN=AX(KTWO)/DY \$ YPN=AY(KTWO)/DY \$ KQ=KTWO-KADD	DRAW	42
430	XPL=AX(KQ)/DY \$ YPL=AY(KQ)/DY	DRAW	43
	IF (ABS(XPN-XPL) .LT. 0.0005 .AND.	DRAW	44
	\$ABS(YPN-YPL) .LT. 0.0005) GO TO 410	DRAW	45
	GO TO 420	DRAW	46
	C POINTS TOO CLOSE TOGETHER	DRAW	47
410	KQ=KQ-KADD	DRAW	48
	GO TO 430	DRAW	49
420	DSC=SQRT((XPN-XPL)**2+(YPN-YPL)**2)	DRAW	50
	CALL PLOT(XPN,YPN,2)	DRAW	51
	XB=WI*(YPN-YPL)/DSC \$ YB=-WI*(XPN-XPL)/DSC	DRAW	52
	C DRAW TICK MARK ON RAY	DRAW	53
	CALL PLOT (XPN+XB,YPN+YB,2)	DRAW	54
	CALL PLOT (XPN-XB,YPN-YB,2)	DRAW	55
	CALL PLOT (XPN,YPN,2)	DRAW	56
		DRAW	57
303	IF (KTWO .EQ. LAST) GO TO 106	DRAW	58
	KTWO=KTWO+KADD	DRAW	59
	GO TO 105	DRAW	60
106	IF (KADD .GE. 0) GO TO 108	DRAW	61
	IF (FAN .NE. 0) GO TO 205	DRAW	62
	CALL NUMBER (AX(1)/DY,AY(1)/DY,0.1,XN,0.0,-1)	DRAW	63
	GO TO 205	DRAW	64
108	IF (FAN .EQ. 0) GO TO 205	DRAW	65
	C NUMBER RAY AT THE TERMINAL POINT	DRAW	66
	CALL NUMBER (AX(KMAX)/DY,AY(KMAX)/DY,0.1,XN,0.0,-1)	DRAW	67
205	RETURN	DRAW	68
	END	DRAW	69

CHAPTER IV USING THE COMPUTER PROGRAM

4.1 Preparation of the Water Depth Grid. Once a coastal area is selected for making wave forecasts a water depth grid must be prepared. Details with numerous illustrations for preparing water depth grids are given by Wilson (1966). It is necessary to obtain charts of the region of interest containing sufficiently detailed bathymetric information.

A water depth grid is rectangular in shape. The value of x varies between 0 and AMM while y varies from 0 to ANN. The values of AMM and ANN are defined by

$$\text{AMM} = \text{MM} - 1 \quad (4-1)$$

$$\text{ANN} = \text{NN} - 1 \quad (4-2)$$

where MM is the number of water depth values in a y -column and NN is the number of columns. The value of MM must be an integral multiple of 16. If another number is preferred the format statement in the computer program used to input the water depth values must be changed. The maximum values of MM and NN depend upon the storage capacity of the computer. In the computer program presented in this report the values of MM and NN are assumed not to exceed 100. If the grid requirements exceed the storage capacity of the computer the coastal region of interest can be divided into several overlapping grids.

The xy -coordinate system is right-handed with the x -axis extending seaward. The direction of the x -axis with respect to true north is defined as CNVRSRA. The use of CNVRSRA makes it possible to define the input and output ray directions with respect to true north.

The distance between water depths in the x - or y -directions is a grid interval or grid unit and is denoted by GRID. This distance must be small enough for the water depth grid to describe adequately the bottom topography. If it is desirable for rays to start in deep water the grid must extend at least several grid units seaward of the deep water depth of the largest wave period of interest. In this report deep water is defined as any depth greater than $0.64 \lambda_d$ where λ_d is the deep water wavelength. This definition of deep water is chosen since the collinear group speed is nearly invariant for greater water depths.

To determine the location of the water depths to be read from a chart lines can be drawn on tracing paper parallel to the x - and y -axes of the grid and separated a

distance equal to a grid unit. The tracing paper is placed on the chart and water depths are estimated for the points defined by the intersection of the grid lines. The water depths can be recorded in any system of units.

One of the program options is to have the shoreline drawn on a plot. In order for the location of the shoreline to be computed it is necessary to determine negative values of water depths for at least two grid points landward of the shoreline. The negative values are determined by drawing the reflection of water depth contours on land with respect to the shoreline. Zero water depths can be used to fill out a column for grid points more than two grid units landward of the shore.

4.2 Preparing a Computer Run. The way in which data is prepared for a computer run is illustrated on the coding form in Figure (4-1). Eight types of computer cards are used. The columns available for each parameter are outlined by rectangles. The positions of decimal points for real numbers are indicated. If there is no decimal point the number is an integer and is placed in the rectangle as far to the right as possible. The input parameters must appear on each card as shown, and the card types must be in the order indicated.

Six computer cards are required to input the data for both S and EM. These numbers are used in the surface fitting routine and they are the same for all computer runs.

For the third type of computer card, MXPLOT is the number of runs for a given operation of the computer program. The PROJCT is a 6-character label of any combination of letters and numbers. The label can be used, for example, to indicate a project number. An alternative use is to identify which water depth grid is used for the run. It appears in both the printed output and on the plot. DATE1 and DATE2 are used to date the run. DATE1 can be used for the year and the month in the form ZZ/YY/. DATE2 can be used for the day in the form XX. The DIR is another 6-character label of any combination of letters and numbers. This label appears only on the plot. One possible label is WAVPAK, which can denote that wave packet trajectories are presented. If the rays have a common initial direction, DIR can be used to indicate that direction.

The number of rays for a given run, NOR, is input on the fourth type of computer card. The values of NPT and SK determine the amount of printed output. If NPT is not zero there is printed output for the first ray point, those points which are an integral multiple of SK, and the last point. If NPT is zero printed output occurs only for the initial and terminal ray points. The value of HT is the length of the y-axis of the plot in inches or centimeters. If CIN is not zero tick marks are placed on the rays at equal intervals of travel time given by the value of CIN in

IBM

FORTRAN Coding Form

PROGRAM NAME	DATE	PROGRAMMING INSTRUCTIONS	GRAPHIC PUNCH	PAGE OF CARD ELECTRIC NUMBER*
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IDENTIFICATION		FORTRAN STATEMENT		IDENTIFICATION	
LINE	CHARACTER	LINE	CHARACTER	LINE	CHARACTER
1	S	1		1	
2		2		2	
3		3		3	
4		4		4	
5		5		5	
6		6		6	
7		7		7	
8		8		8	
9		9		9	
10		10		10	
11		11		11	
12		12		12	
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51		51		51	
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67		67		67	
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69		69		69	
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81		81		81	
82		82		82	
83		83		83	
84		84		84	
85		85		85	
86		86		86	
87		87		87	
88		88		88	
89		89		89	
90		90		90	
91		91		91	
92		92		92	
93		93		93	
94		94		94	
95		95		95	
96		96		96	
97		97		97	
98		98		98	
99		99		99	
100		100		100	

*Number of turns per card may vary slightly

Figure (4-1). FORMAT OF THE INPUT PARAMETERS

seconds. If no tick marks are desired CIN is zero.

The x- and y-axes of the plot will be calibrated and labeled if NAX is not zero. If NAX is zero the plot borders are drawn but the axes are not calibrated. The shoreline is drawn on the plot if NSH is not zero. If the shoreline is not desired NSH must be zero. The number of sounding water depths for a plot is NCO. There cannot be more than 9 sounding depths, and they are input on the CONTUR computer card. If NCO is zero there are no sounding depths; in this case the CONTUR card must be removed from the input. If a water depth grid is to be read in the input for the computer run the value of NXCMAT is zero. If NXCMAT is not zero the depth grid for the previous run is used again. This situation can arise if MXPLOT is greater than one.

The fifth type of computer card contains the input dimensions for the water depth grid. A description of the quantities MM, NN, CNVRS, and GRID are described in the previous section. The angle CNVRS is given in degrees and GRID is given in feet or meters. The value of DCON is chosen so that the product of DCON and a water depth in CMAT yields a value with units of feet or meters. If English units are to be used in the input and output MOE is zero. If MOE is not zero Metric units are used. The value of NNSKIP is the amount by which y is incremented in selecting columns for locating sounding water depths. For example, if NNSKIP is 15 and NN is 64, sounding water depths are located for the 2, 17, 32, 47, and 62 y-columns.

The sixth type of computer card is used to input the water depth grid (CMAT). The units of CMAT determine the value of DCON. There are 16 water depths on each card. The water depths are entered column by column starting with the first column. There are NN columns. In each column the water depths are entered starting with the land values, if any, and proceeding seaward. There are MM values per column. The format for entering the water depths does not include numbers beyond the decimal points. Near shore it may be desirable to record water depths to the nearest tenth of a foot or meter. On some computer systems it is possible to enter data routinely in this form with the indicated format for CMAT being overridden. If such a capability is not available it may be desirable to alter the format statement for CMAT in MAIN.

If NCO is not zero the CONTUR computer card is used to input the sounding water depths in feet or meters. The number of sounding depths must agree with NCO which should not exceed 9. If NCO is zero the CONTUR card must be removed.

The eighth type of computer card is used to input the particulars for each ray. There must be as many ray cards as declared in the input for NOR. The initial time step interval between ray points in seconds is DELTAT. The wave period in seconds is TT, and X, Y are the initial ray

coordinates.¹ The initial wave packet and wavelet directions are A and AV, respectively. The directions are in degrees and are the directions from which the waves come with respect to true north. The initial wave height in feet or meters is HGTZ.

The friction factor is CF. The value of KRTOL determines the accuracy of the calculations of the refraction coefficient with the exception of near reflection points. As a general rule, if accuracy is required to the second decimal point KRTOL is 0.01. If accuracy is desired to the third decimal point KRTOL is 0.001.

To continue a ray beyond a reflection point ROP is set unequal to zero. If ROP is zero a ray is stopped at a reflection point. A test is made to determine if a wave breaks if WBCOP is not zero. If WBCOP is zero there is no test to determine if a wave breaks. If the ray is to be numbered at its terminal point FAN is set unequal to zero. A group of rays should be numbered at their terminal points if they have a common origin. If FAN is zero the ray is numbered at its initial point.

A sample of input data for a computer run with 6 rays is shown in Figure (4-2). Since the water depth contours are parallel, only one of the 64 columns of water depth values is shown in the rectangle labeled CMAT. The computer output for this run is presented in Section (4.6). Therefore, if desired, these input data can be used to check the computer program.

4.3 The Printed Output when $NPT \neq 0$. The most detailed computer printout is obtained when $NPT \neq 0$. The first thing that occurs in the printout is the page heading. This contains the PROJCT, date, plot number, ray period, ray number, input time step, friction factor, and KRTOL. If at the first ray point this is followed by a statement denoting whether English or Metric units are used in the output. Further, the initial value of the time derivative of the ray separation factor is given. The column headings appear next in the output. Beyond the first point of a ray the page and column headings occur after every 60 lines of additional printout.

The column headings identify the ray particulars which appear in the output. They contain the ray point number MAX, the ray coordinates X, Y, and the water depth H in meters or feet. The wave packet and wavelet directions are denoted, respectively, by PACK and WAVE. These are the directions in degrees from which the waves come with respect to true north. The distance in grid units between ray points is given by D, and FK is the packet ray curvature in radians per grid unit. The angle in degrees by which the x'y'-coordinate system is rotated with respect to the positive X-axis for computations is given by ALFA. The geometric group speed, collinear group speed, and phase speed are denoted, respectively, by G, U,

¹The initial ray points should be at least two grid units from a grid boundary.

FORTRAN STATEMENT										IDENTIFICATION SEQUENCE									
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2.22488	-1.59689	-1.59689	-1.59689	0.25	0.25	0.47	36842	0.25											
-1.59689	1.689593	1.036184	-0.375	-0.375		-0.31	57895	-0.1875											
-1.59689	1.036184	1.689593	-0.1875	-0.1875		-0.31	57895	-0.375											
0.25	-0.375	-0.1875	0.125	0.125		0.0		0.0625											
0.4736842	-0.3157895	-0.3157895	0.0	0.0		0.21	05263	0.0											
0.25	-0.1875	-0.375	0.0625	0.0625		0.0		0.125											
1.1.1.1.1.1.1.1.1.1.1.																			
0.0.1.1.1.1.2.2.2.3.3.																			
1.2.0.1.2.3.0.1.2.3.1.2.																			
0.0.1.1.1.1.4.4.4.9.9.																			
0.0.0.1.2.3.0.2.4.6.3.6.																			
1.4.0.1.4.9.0.1.4.9.1.4.																			
1 PAR 2 78/03/ 01	WAVPAK																		
6 1 5	15.24	300.0				1	1	4											
64 64 180.0	750.0	1.0				1	15												
-15.0	-7.5	0.0	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	75.0	82.5	90.0	97.5				
105.0	112.5	120.0	127.5	135.0	142.5	150.0	157.5	165.0	172.5	180.0	187.5	195.0	202.5	210.0	217.5				
225.0	232.5	240.0	247.5	255.0	262.5	270.0	277.5	285.0	292.5	300.0	307.5	315.0	322.5	330.0	337.5				
345.0	352.5	360.0	367.5	375.0	382.5	390.0	397.5	405.0	412.5	420.0	427.5	435.0	442.5	450.0	457.5				
CMAT																			
100.0	200.0	300.0	400.0																
25.0	20.0	4.0	60.0	15.0		15.0		15.0		3.0	0.1	0.001		1	1	0			
25.0	20.0	4.0	60.0	23.0		23.0		23.0		3.0	0.1	0.001		1	1	0			
25.0	12.0	25.0	60.0	165.0		165.0		165.0		3.0	0.1	0.001		0	1	0			
25.0	12.0	25.0	60.0	120.0		120.0		120.0		3.0	0.1	0.001		0	1	0			
25.0	17.0	42.0	3.0	200.0		200.0		200.0		3.0	0.0	0.001		0	0	1	0		
25.0	17.0	42.0	3.0	220.0		220.0		220.0		3.0	0.0	0.001		0	0	1	0		

Figure (4-2). SAMPLE INPUT DATA FOR A COMPUTER RUN

and V. The units are in meters/second or feet/second. The wave height is HGT in meters or feet. The shoaling coefficient is identified by KS, and KF represents the friction coefficient. The number of breakups, if any, of the initial time step is NO. The refraction coefficient is defined by KR. An estimate of how well the computed water bottom surface fits the actual water depths is given by PCTDIF (see the NOTATION). The smaller the value the better the fit. The units of H, HGT, G, U, and V are identified in the output.

The travel time along the wave packet trajectory does not appear in the output. However, it can be determined by subtracting one from MAX and multiplying the result by the input time step.

The ray particulars are printed out for the first ray point, those points which are an integral multiple of SK, and the last point. Printout occurs for a reflection point should one occur. Note that the number of ray points, NO, which occur when there is a breakup of the input time step interval is not counted in MAX. There is no printout for ray points which occur within a breakup.

If there is printed output and a ray is near a reflection point (Sections (2.3), b, (2.5), d, (3.8), and 3.9)) an asterisk appears in the printout to the left of the value of FK. If there is printed output when there is a breakup of the input time step interval due to the calculations of the refraction coefficient (Sections (2.5), c and (3.9)) an asterisk appears in the printout to the left of the value of KR. If there is no printout ($SK > 1$) the asterisk appears with the ray particulars for the next ray point where printout occurs. Only the asterisk with FK appears in the next printout should both conditions for an asterisk be satisfied at preceding ray points where there is no printed output. If the ray is at a reflection point (Sections (2.3), b and (3.8)) an asterisk occurs in the printout to the left of the value of MAX.

A number of descriptive printouts appear with the ray particulars when certain types of calculations occur or when a ray terminates. If the ray curvature of the wave packet is averaged in computing a ray point (Section (3.8)) the following printout appears.

(1) PACKET CURVATURE AVERAGED

If $SK = 1$ this descriptive printout follows the ray particulars. If $SK > 1$ the descriptive printout occurs even if there is no printed output of the ray particulars. In this case, the curvature is averaged for a ray point between the ray points preceding and following the descriptive printout.

When there is a reflection one of three descriptive printouts occurs (Section (2.3), b).

(2) REFLECTION: SNELLS LAW WITH PHASE VELOCITY

(3) REFLECTION: PACKET CURVATURE ITERATION NOT CONVERGING

(4) REFLECTION: NEAR REFLECTION POINT

The ray point where one of these three descriptive printouts occurs is the last ray point if ROP = 0.

When a ray terminates one of the following descriptive printouts can appear in the output.

(5) DIMENSION OF OUTPUT-ARRAYS EXCEEDED

(6) RAY REACHED SHORE

(7) RAY REACHED GRID BOUNDARY

(8) PACKET CURVATURE ITERATION NOT CONVERGING

(9) CAUSTIC OR FOCAL POINT

(10) WAVE BREAKS

(11) REFLECTION HANG-UP

(12) BREAKUP TIME STEP LESS THAN 0.5 SECOND

Printout (5) occurs if the sum of the number of ray points and tick marks is equal to or greater than the array dimension MMAX. Printout (6) is obtained if the water depth becomes zero or negative. Printout (7) results if the ray point is within 1.5 grid units of a grid boundary. The conditions for a reflection point are not satisfied if Printout (8) occurs. Printout (9) is produced if the ray separation factor becomes zero or negative. The condition for Printout (10) is given in Section (2.7). Printout (11) is obtained if there are successive reflections at the same ray point. Printout (12) can occur if the calculation time step becomes too small in either calculating the ray path near a reflection point (Section (2.3), b) or in calculating the ray separation factor (Section (2.5), c).

4.4 The Printed Output when NPT = 0. When NPT = 0 there is printed output at only the first and last ray points. The page heading contains PROJCT, the date, and plot number. A statement signifies whether English or Metric units are employed. The column headings define the ray number N, the wave period, MAX, X, Y, H, PACK, WAVE, HGT, the input time step, the friction factor, and KRTOL. All of these ray particulars appear in the printed output at the first ray point. At the last ray point the input time step, the friction factor, and KRTOL are not repeated. In their place is a descriptive statement which explains why the ray

terminated.

When $NPT = 0$, Printout (1) does not appear. There is no descriptive printout for a reflection unless the reflection occurs at the last ray point ($ROP = 0$). Then the following descriptive printout occurs.

(13) REFLECTION

The remaining descriptive printouts are the same as discussed in Section (4.3).

4.5 The Plots. Each plot contains a label consisting of PROJCT, the date, the scale factor, the time in seconds between tick marks on a ray, if any, the plot number, and DIR. If $NAX \neq 0$ the axes of the plot are calibrated and labeled. If $NSH \neq 0$ the shoreline is drawn. If $NCO \neq 0$ sounding water depth values are labeled. Each ray is numbered. If $FAN = 0$ the number appears at the initial ray point, and if $FAN \neq 0$ the ray is numbered at its terminal point.

4.6 Examples of Computer Output. Figures (4-3) through (4-8) show the printed output for the 6 rays of the sample input data presented in Figure (4-2). The plot of the rays is shown in Figure (4-9). The examples illustrate rays beginning both at an intermediate water depth and in deep water. Three different wave periods are considered. The second ray undergoes a reflection (Section (2.1), b and Section (2.5), d). For the last two rays the friction factor is assumed to be zero. Tick marks and sounding water depths are shown on the plot.

Figure (4-10) shows a portion of the Gulf of Mexico off the southwestern Florida coast. A water depth grid was prepared for this region with $GRID = 14886.2$ feet (4.537 km) and $CNVRSA = 180^\circ$. A ray plot for this region is shown in Figure (4-11). Figure (4-12) contains the printed output for this plot when $NPT = 0$. The first two rays start at an intermediate water depth, whereas the remaining rays begin in deep water. Figure (4-13) displays printed output for the first portion of ray number 1. Since the water depth contours are not parallel there is a variation in ALFA.

Ray number 2 has a reflection. Figure (4-14) shows a listing of the ray particulars near the reflection point. The wave packet and wavelet angles in the xy - and $x'y'$ -coordinate systems are defined by Equations (3-1), (3-2), (2-26), and (2-27). At the reflection point the angles in the xy -coordinate system are $\theta_c = 274.86^\circ$ and $\gamma_c = 1.94^\circ$. In the $x'y'$ -coordinate system $\theta' = 2.41^\circ$ and $\gamma' = 89.49^\circ$.

Ray number 12 illustrates the importance of examining the ray particulars in the printout. Figure (4-15) shows the printed output for this ray. A message in the output states there is a reflection. However, a reflection is not

likely since H , θ' , γ' , FK , and G do not exhibit the behavior characteristic of a reflection. At the ray point where the reflection is indicated $\theta' = 115.78^\circ$ and $\gamma' = 117.52^\circ$. This false reflection is the result of a large change in $ALFA$ between successive ray points. When this occurs the water depth grid is not sufficiently detailed to adequately represent the changing water depth contours.


```
PROJECT NO. PAR 2, 78/J3/01, PLOT NO. 1, PERIOD= 20.0SEC., RAY NO. 1, DELTAT= 25.00, CF= .100000, KRYOL= .001000
THE OUTPUT IS IN METRIC UNITS. H-4GT(METER). G,U,V(METFR/SECOND). D(BETA)/DT = -2.201E-04
```

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Figure (4-3). LISTING FOR RAY NUMBER 1 OF SAMPLE INPUT DATA

[illegible]

PROJECT NO. PAR 2, 78/33/01, PLOT NO. 1, PERIOD= 12.0 SEC., RAY NO. 3, DELTAT= 25.00, CF= .100000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS. H, HGT(METER). G, U, V(METER/SECOND). D(BETA)/DT = -0.

MAX X	Y	H	PACK	WAVE	D	FK	ALFA	G	U	V	HGT	KS	KF	NO KR	PCTDIF
15	25.379	60.087	172.443	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	23.279	59.277	153.117	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	20.279	58.465	143.477	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	17.279	57.653	133.837	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	14.279	56.841	124.197	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	11.279	56.029	114.557	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	8.279	55.217	104.917	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	5.279	54.405	95.277	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	2.279	53.593	85.637	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.279	52.781	75.997	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	52.000	66.357	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	51.217	56.717	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	50.429	47.077	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	49.641	37.437	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	48.853	27.797	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	48.065	18.157	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	47.277	8.517	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	46.489	-1.123	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	45.701	-10.483	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	44.913	-19.843	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	44.125	-29.203	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	43.337	-38.563	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	42.549	-47.923	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	41.761	-57.283	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	40.973	-66.643	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	40.185	-76.003	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	39.397	-85.363	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	38.609	-94.723	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	37.821	-104.083	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	37.033	-113.443	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	36.245	-122.803	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	35.457	-132.163	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	34.669	-141.523	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	33.881	-150.883	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	33.093	-160.243	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	32.305	-169.603	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	31.517	-178.963	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	30.729	-188.323	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	29.941	-197.683	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	29.153	-207.043	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	28.365	-216.403	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	27.577	-225.763	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	26.789	-235.123	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	26.001	-244.483	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	25.213	-253.843	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	24.425	-263.203	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	23.637	-272.563	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	22.849	-281.923	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	22.061	-291.283	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	21.273	-300.643	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	20.485	-310.003	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	19.697	-319.363	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	18.909	-328.723	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	18.121	-338.083	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	17.333	-347.443	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	16.545	-356.803	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	15.757	-366.163	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	14.969	-375.523	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	14.181	-384.883	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	13.393	-394.243	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	12.605	-403.603	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	11.817	-412.963	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	11.029	-422.323	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	10.241	-431.683	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	9.453	-441.043	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	8.665	-450.403	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	7.877	-459.763	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	7.089	-469.123	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	6.301	-478.483	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	5.513	-487.843	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	4.725	-497.203	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	3.937	-506.563	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	3.149	-515.923	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	2.361	-525.283	165.000	165.000	0.000	-.02	9.37	9.37	18.74	3.000	1.000	1.000	1.000	.01
15	0.000	1.573	-534												

PROJECT NO. PAR 2, 78/03/01, PLOT NO. 1, PERIOD= 12.0SEC., RAY NO. 4, DELTAT= 25.00, CF= .100000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS. G,U,V (METER/SECOND).
H,HGT (METER). D(BETA)/DT = -0.

[illegible]

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Figure (4-6). LISTING FOR RAY NUMBER 4 OF SAMPLE INPUT DATA


```
PROJECT NO. PAR 2, 78/03/01
, PLOT NO. 1, PERIOD= 17.0SEC., RAY NO. 6, DELTAT= 25.00, CF=0.000000, KRTOL= .001000
```

THE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V(METER/SECOND).
 $D(BETA)/DT = -0.$

[illegible]

THIS IS THE END.

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Figure (4-8). LISTING FOR RAY NUMBER 6 OF SAMPLE INPUT DATA

PROJ. NO. PAR 2, 78/03/01
 SCL = 1/310039, CIN = 300
 PLOT NO. 1, DIR. = WAVPAK

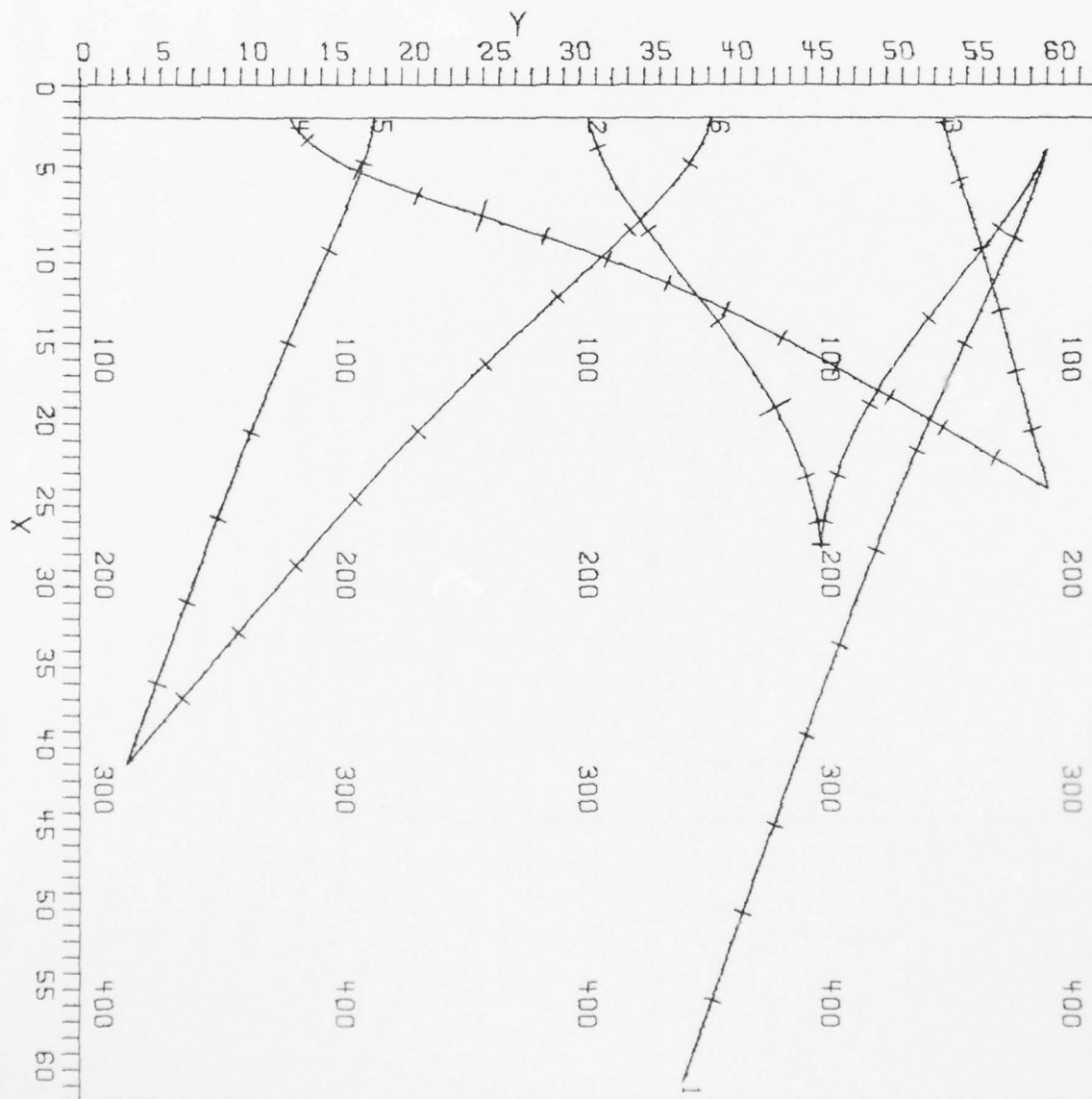


Figure (4-9). PLOT FOR SAMPLE INPUT DATA

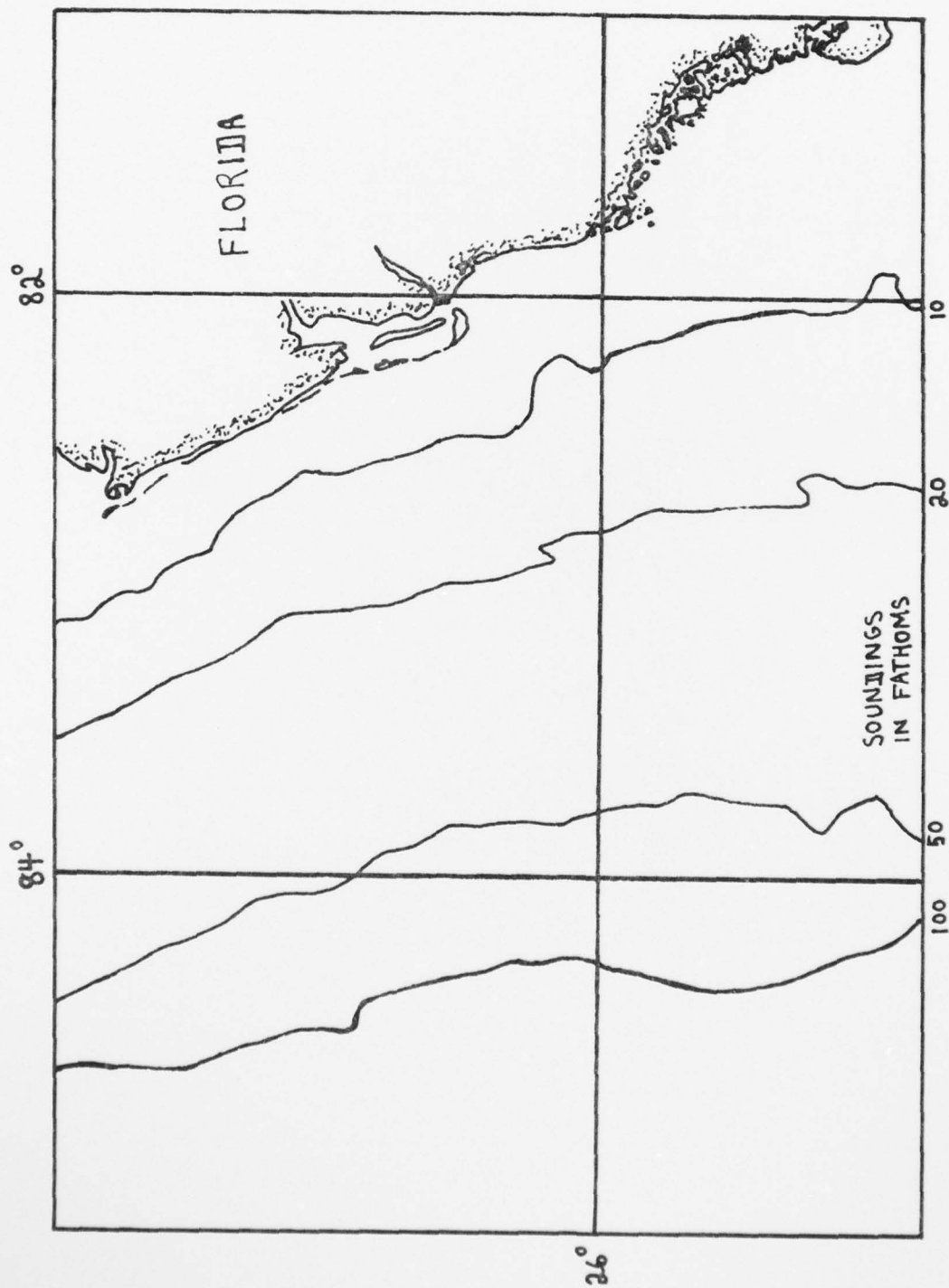


Figure (4-10). GULF OF MEXICO OFF THE SOUTHWESTERN FLORIDA COAST

PROJ. NO. GOM 3, 78/03/01
 SCL = 1/3179692, CIN = 0
 PLOT NO. 2, DIR. = WAVPAK

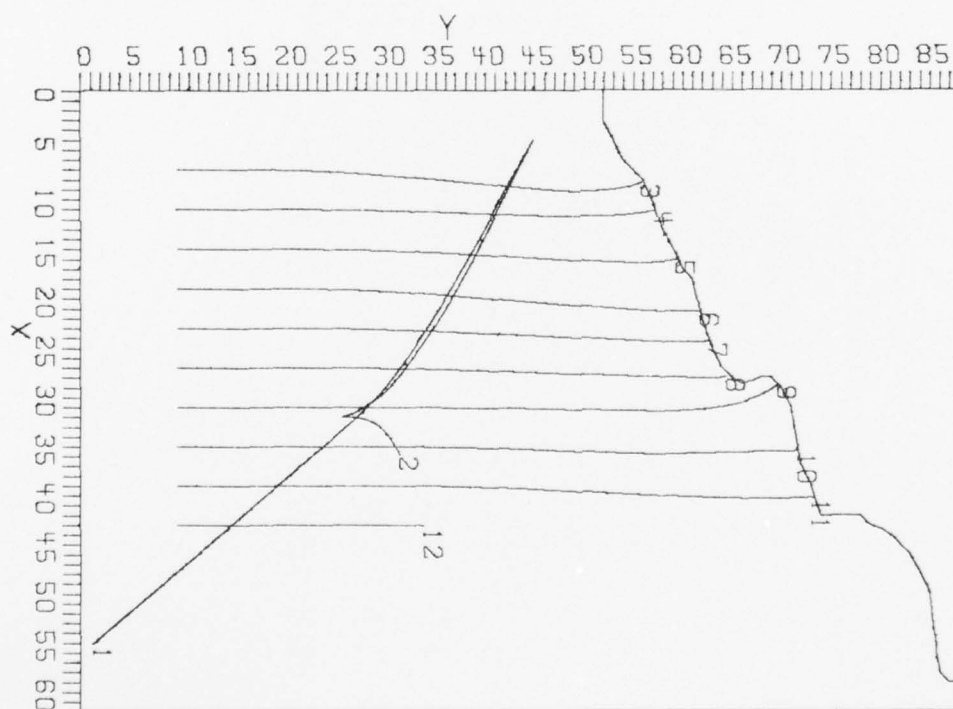


Figure (4-11). PLOT OF RAYS OFF THE SOUTHWESTERN
 FLORIDA COAST

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PROJECT NO. GOM 3, 78/03/51 , PLOT NO. 2										
THE OUTPUT IS IN ENGLISH UNITS. H,HGT(Feet). G,U,V(FEET/SECOND).										
N	PERIOD	MAX	X	Y	H	PACK	WAVE	HGT	DELTA OF	KRIOL
1	12.0	1	5.00	46.00	74.74	30.00	30.00	5.0000	50.00	.005000
1	12.0	736	56.12	1.44	109.18	49.23	10.06	5.0781		.001000 RAY REACHED GRID BOUNDARY
2	12.0	1	5.00	46.00	74.74	28.00	28.00	5.0000	50.00	.005000
2	12.0	572	36.78	32.50	234.50	339.42	340.50	3.4110		.001000 BREAKUP TIME STEP LESS THAN 0.5 SECOND
3	12.0	1	5.00	46.00	74.74	27.00	27.00	5.0000	50.00	.005000
3	12.0	427	9.32	57.00	9.70	27.29	243.51	4.6359		.001000 RAY REACHED SHORE
4	12.0	1	12.00	10.00	779.84	270.00	270.00	5.0000	50.00	.005000
4	12.0	434	12.14	58.21	5.87	254.32	249.39	4.9579		.001000 WAVE BREAKS
5	12.0	1	16.00	10.00	846.32	270.00	270.00	5.0000	50.00	.005000
5	12.0	455	17.07	60.45	7.25	253.75	249.48	5.3563		.001000 WAVE BREAKS
6	12.0	1	20.00	10.00	877.93	270.00	270.00	5.0000	50.00	.005000
6	12.0	477	22.32	62.38	6.43	268.84	266.49	5.3723		.001000 WAVE BREAKS
7	12.0	1	24.00	10.00	1012.37	270.00	270.00	5.0000	50.00	.005000
7	12.0	487	25.31	63.73	6.44	259.49	266.76	5.2451		.001000 WAVE BREAKS
8	12.0	1	28.00	10.00	1188.82	270.00	270.00	5.0000	50.00	.005000
8	12.0	517	28.86	65.33	5.27	244.79	241.54	4.9792		.001000 WAVE BREAKS
9	12.0	1	32.00	10.00	1620.58	270.00	270.00	5.0000	50.00	.005000
9	12.0	573	29.72	70.73	1.83	227.40	226.14	1.5455		.001000 WAVE BREAKS
10	12.0	1	36.00	10.00	2375.57	270.00	270.00	5.0000	50.00	.005000
10	12.0	576	36.47	72.78	5.44	266.80	264.53	4.5235		.001000 WAVE BREAKS
11	12.0	1	40.00	10.00	3096.22	270.00	270.00	5.0000	50.00	.005000
11	12.0	585	41.15	74.38	6.05	257.92	252.36	4.9206		.001000 WAVE BREAKS
12	12.0	1	44.00	10.00	3720.80	270.00	270.00	5.0000	50.00	.005000
12	12.0	239	44.06	34.91	232.41	271.22	269.48	4.7737		.001000 REFLECTION

THIS IS THE END.

Figure (4-12). LISTING OF RAYS WHEN NPT = 0

X1

Figure (4-13). LISTING OF RAY MOVING TOWARD DEEP WATER

PROJECT NO. 604 3, 7803/01, PLOT NO. 1, PERIOD=12.0 SEC., RAY NO. 2, DELTA= 50.00, CF= .005000, KRTOL= .001000

MAX X	Y	H	PACK	HAVE	D	FK	ALFA	G	U	V	HST	KS	KF	NO KR	PCTOIF
57	2	0	23	39	50	39	39	39	39	39	39	39	39	39	39
58	3	0	23	39	50	39	39	39	39	39	39	39	39	39	39
59	4	0	23	39	50	39	39	39	39	39	39	39	39	39	39
60	5	0	23	39	50	39	39	39	39	39	39	39	39	39	39
61	6	0	23	39	50	39	39	39	39	39	39	39	39	39	39
62	7	0	23	39	50	39	39	39	39	39	39	39	39	39	39
63	8	0	23	39	50	39	39	39	39	39	39	39	39	39	39
64	9	0	23	39	50	39	39	39	39	39	39	39	39	39	39
65	10	0	23	39	50	39	39	39	39	39	39	39	39	39	39
66	11	0	23	39	50	39	39	39	39	39	39	39	39	39	39
67	12	0	23	39	50	39	39	39	39	39	39	39	39	39	39
68	13	0	23	39	50	39	39	39	39	39	39	39	39	39	39
69	14	0	23	39	50	39	39	39	39	39	39	39	39	39	39
70	15	0	23	39	50	39	39	39	39	39	39	39	39	39	39
71	16	0	23	39	50	39	39	39	39	39	39	39	39	39	39
72	17	0	23	39	50	39	39	39	39	39	39	39	39	39	39
73	18	0	23	39	50	39	39	39	39	39	39	39	39	39	39
74	19	0	23	39	50	39	39	39	39	39	39	39	39	39	39
75	20	0	23	39	50	39	39	39	39	39	39	39	39	39	39
76	21	0	23	39	50	39	39	39	39	39	39	39	39	39	39
77	22	0	23	39	50	39	39	39	39	39	39	39	39	39	39
78	23	0	23	39	50	39	39	39	39	39	39	39	39	39	39
79	24	0	23	39	50	39	39	39	39	39	39	39	39	39	39
80	25	0	23	39	50	39	39	39	39	39	39	39	39	39	39
81	26	0	23	39	50	39	39	39	39	39	39	39	39	39	39
82	27	0	23	39	50	39	39	39	39	39	39	39	39	39	39
83	28	0	23	39	50	39	39	39	39	39	39	39	39	39	39
84	29	0	23	39	50	39	39	39	39	39	39	39	39	39	39
85	30	0	23	39	50	39	39	39	39	39	39	39	39	39	39
86	31	0	23	39	50	39	39	39	39	39	39	39	39	39	39
87	32	0	23	39	50	39	39	39	39	39	39	39	39	39	39
88	33	0	23	39	50	39	39	39	39	39	39	39	39	39	39
89	34	0	23	39	50	39	39	39	39	39	39	39	39	39	39
90	35	0	23	39	50	39	39	39	39	39	39	39	39	39	39
91	36	0	23	39	50	39	39	39	39	39	39	39	39	39	39
92	37	0	23	39	50	39	39	39	39	39	39	39	39	39	39
93	38	0	23	39	50	39	39	39	39	39	39	39	39	39	39
94	39	0	23	39	50	39	39	39	39	39	39	39	39	39	39
95	40	0	23	39	50	39	39	39	39	39	39	39	39	39	39
96	41	0	23	39	50	39	39	39	39	39	39	39	39	39	39
97	42	0	23	39	50	39	39	39	39	39	39	39	39	39	39
98	43	0	23	39	50	39	39	39	39	39	39	39	39	39	39
99	44	0	23	39	50	39	39	39	39	39	39	39	39	39	39
100	45	0	23	39	50	39	39	39	39	39	39	39	39	39	39

Figure (4-14). LISTING OF RAY PARTICULARS NEAR A REFLECTION POINT

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NOTATION

The principal symbols are defined. The expressions in parentheses are alternate representations of the symbol being defined.

A	(ANGLE, PACK, SVA) The program symbol for θ . In the input and output A is the direction in degrees from which the wave packet comes with respect to true north. Internally in the program A is the direction in radians with which the wave packet moves with respect to the positive x-axis.
AA	The wave packet direction at the new ray point.
AAA, AAAV	The average of the values at the new and previous ray points, respectively, of the wave packet and wavelet directions.
ABAR	The average of the wave packet directions at the present and new points.
ALFA	(SVALFA, PALFA) The program symbol for α .
AMM, ANN	The maximum values of x and y, respectively, for a water depth grid.
AV	(AAV, GAM, WAVE, SAV, SVAV) The program symbol for γ . The directions of AV are defined following the same conventions used in the definitions of A.
AVP	(GP) The program symbol for γ' .
AX, AY	The arrays used to store the locations of ray points and tick marks.
a	A constant for a given wave period.
BDZ	(SBDZ) The program symbol for $d\beta/dt$.
BDZ5	The fifth order Runge-Kutta solution of $d\beta/dt$.
BRK	Is zero except during a breakup of the time step interval when the value is one. After returning to MOVE from HEIGHT, the value of BRK determines where the program resumes.

BZ	(SBZ) The program symbol for β .
BZTOL	The limiting value for $ \text{EBZ} $ and $ \text{EBDZ} $ in the Runge-Kutta calculations of β and $d\beta/dt$. If $ \text{EBZ} $ or $ \text{EBDZ} $ exceeds or is equal to BZTOL the time step interval is halved.
BZ5	The fifth order Runge-Kutta solution of β .
b	A constant for a given wave period.
C	An array of 12 water depths from CMAT used to fit a quadratic surface in the vicinity of the ray point.
CF	The program symbol for c_f .
CIN	If CIN is not zero, its value is the travel time between tick marks along a ray. In the input CIN is in seconds, but for the calculations CIN is converted to hours. If CIN is zero there are no tick marks on a ray.
CMAT	The water depth grid in a two dimensional array.
CNVRSA	The direction of the positive x-axis of the water depth grid with respect to true north. The use of this conversion angle permits the ray directions to be defined with respect to true north in the input and output.
COL	If COL is not zero the plotter will pause before the ray is plotted. If COL is zero the plotter does not pause.
CONTUR	An array containing the sounding water depths in feet or meters. There can be as many as 9 values.
CORI	A conversion factor used in calculating SCLI.
C_s	The constant of Snell's law.
c_f	The friction factor.
D	(PD) The incremental distance in grid units between ray points.
DADX	The program symbol for $d\theta/dx'$.
DATE1, DATE2	The year, month, and day.

DAVDX	The program symbol for dy/dx' .
D(BETA)/DT	The initial value of $d\beta/dt$.
DCON	A conversion factor to convert the water depths in CMAT to feet or meters.
DELA	The change in the wave packet direction from the present to the new ray point.
DELTAT	(SDLTAT) The time step interval in seconds between ray points.
DELX, DELY	The change in the values from the present to the new ray points of the x and y coordinates, respectively.
DEP	(SDEP, PDEP) The program symbol for h.
DGDY	(SADGDY) The program symbol for dG/dx' .
DGDXX	The program symbol for $d^2G/(dx')^2$.
DHDX	The program symbol for dh/dx' .
DHDXX	The program symbol for $d^2h/(dx')^2$.
DIR	A label of 6 letters and numbers used to identify a plot.
DN	An expression used to compute W.
DPHIDX	The program symbol for $d\phi/dx'$.
DRHODX	The program symbol for dp/dx' .
DSIGDX	The program symbol for $d\sigma/dx'$.
DUD	The ratio of the phase speed at the present ray point to the value at the previous ray point.
DVDX	The program symbol for dv/dx' .
DVDXX	The program symbol for $d^2v/(dx')^2$.
DUDX	The program symbol for dU/dx' .
DUDXX	The program symbol for $d^2U/(dx')^2$.
DY	The number of grid units per inch or centimeter for a particular plot.

E	An array of 6 coefficients of the quadratic surface equation which is fitted to the 12 water depths in the array C.
EBDZ	The program symbol for $\epsilon_{\beta t}$.
EBZ	The program symbol for ϵ_{β} .
EM	A two dimensional array of numbers used in computing the array E.
E	The wave energy per unit area.
F	An expression used in the derivation of the friction coefficient.
FAN	If FAN is zero the rays are numbered at their initial points. When FAN is not zero the rays are numbered at their terminal points.
FK	(SVFKB, SAVFK, PFK) The program symbol for κ_G . It is measured in radians/grid unit.
FKBAR	(SVFKB) The average of the ray curvature at the present and new ray points.
FKK	The value of κ_G in radians/grid unit at the new ray point.
FLAGR	Is set equal to zero in MAIN at the beginning of a ray. The value is changed to one in MOVE if there is a reflection and the ray is continued. If FLAGR is one, in HEIGHT the value of ROP is set equal to zero.
FLAG1	If FLAG1 is zero the wavelet direction is computed and the test for total reflection is made in SURFCE. If FLAG1 is not zero these calculations are not made.
FLAG2	If there is total reflection due to the wavelets FLAG2 is set equal to one. Otherwise, FLAG2 is set equal to zero. The reflection test is made in SURFCE.
FLAG3	In MAIN, FLAG3 is set equal to zero at the beginning of a ray. FLAG3 is not used in the program, and it is available for checks or modifications to the program.

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FLORIDA STATE UNIV TALLAHASSEE DEPT OF OCEANOGRAPHY F/G 8/3
A METHOD FOR CALCULATING WAVE PACKET TRAJECTORIES AND WAVE HEIG--ETC(U)
MAR 78 J E BREEDING, K C MATSON, N RIAHI N00014-77-C-0329

UNCLASSIFIED

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FUD	An index used to determine when to write page and column headings depending upon the number of lines of printout.
f	The frequency of the wave ($1/T$).
F	The average rate of energy transmission of the waves.
G	(ZCXY, SG, SVG, PG) The geometric group speed.
GRID	The number of feet or meters per grid unit for a particular water depth grid.
GZERO	The value of the geometric group speed at the first ray point.
g	The acceleration due to gravity.
H	In the theory H is the wave height. In the printed output H represents the water depth.
HGT	(PHGT) The program symbol for the wave height.
HGTZ	The initial value of HGT.
HT	The length of the y-axis in inches or centimeters for a given plot.
HX	The program symbol for $\partial h / \partial x$.
HXX	The program symbol for $\partial^2 h / \partial x^2$.
HXY	The program symbol for $\partial^2 h / \partial x \partial y$.
HY	The program symbol for $\partial h / \partial y$.
HYY	The program symbol for $\partial^2 h / \partial y^2$.
h	The water depth.
I	A variable depending on the ratio of water depth to phase speed.
IFLG	When IFLG is zero a check is made in HEIGHT to determine if there should be a breakup of the time step interval in order to maintain the desired accuracy in the calculations of either β or the ray path. If there is a division of the time step interval, IFLG is set equal to one once the time step interval is sufficiently reduced. When IFLG equals

one further checks for a breakup of the time step interval are not made at new ray points until the breakup ends and calculations are resumed with the initial time step.

INUM	An index to count the ray points within the broken interval when there is a division of the initial time step interval.
j	An index to number ray points.
K	Expressions used in the Runge-Kutta calculations of β and $d\beta/dt$.
KCIN	The number of tick marks along a ray which do not coincide with ray points.
KF	In the program calculations KF is an expression used to evaluate K_F . In the printed output KF is a label for the values of K_F .
KFC	(PKF) The program symbol for K_F .
KMAX	The same as MAX except in DRAW where it is the sum of MAX and KCIN.
KR	(PKR) The program symbol for K_R .
KREST	The number of tick marks along a ray.
KRTOL	Determines the accuracy in the Runge-Kutta calculations of the refraction coefficient. BZTOL depends upon KRTOL.
KS	(PKS) The program symbol for K_S .
K_F	The friction coefficient.
K_R	The refraction coefficient.
K_S	The shoaling coefficient.
k	The wave number $2\pi/\lambda$.
L	Expressions used in the Runge-Kutta calculations of β and $d\beta/dt$.
LI	When NPT is not zero LI is used to determine the number of lines of printout between page and column headings.
LII	When NPT is zero LII is used to determine the number of lines of printout between page and column headings.

ℓ	The perpendicular distance between rays.
MAX	(MAXQ, KMAX) An index to number points along a ray at time intervals equal to the initial time step.
MIT	If MIT is 1 the wave packet curvature approximations in MOVE converge to one value. If MIT is 2 the curvature approximations converge to two values. If the curvature approximations do not converge and there is no reflection MIT is 3. If MIT is 4 a caustic or focal point is computed in HEIGHT. MIT is 5 if it is determined in HEIGHT that the wave breaks. When there is a reflection but the ray is not continued MIT is 6. If MIT is 7 more than one reflection from the same point is determined in MOVE. If MIT is 8 the breakup time step determined in HEIGHT is less than 0.5 seconds.
MM	The dimension of x for a particular water depth grid.
MMAX	The dimension of the AX and AY arrays.
MOE	If MOE is zero the input and output are in English units. If MOE is not zero the input and output are in Metric units.
MXPLOT	The number of runs and the number of plots for a given operation of the computer program.
N	The ray number.
NAX	If NAX is zero the plot has borders but the x- and y-axes are not calibrated. If NAX is not zero the x- and y-axes are calibrated and labeled.
NCO	The number of sounding water depths for a plot. The values are stored in the CONTUR array. The number of sounding depths cannot exceed 9. If NCO is zero there are no sounding depths for the plot.
NDP	The water depth is determined in SURFCE. If the value is greater than 0, NDP is 1 (initialized in RAYN). If the water depth equals or is less than 0, NDP is 2.

NFK The value of NFK is determined in SURFCE. If the ratio of the water depth to the deep water wavelength is greater than 0.64, NFK is 1. Otherwise, NFK is 2.

NGO The value of NGO is determined in MOVE. If a ray point lies within one and one half grid units of a grid boundary NGO is 2. Otherwise, NGO is 1. (Initialized in RAYN.)

NN The dimension of y for a particular water depth grid.

NNSKIP The amount by which y is incremented in selecting columns for locating sounding water depths.

NO (NUMT, KNUMT) The number of divisions when there is a breakup of the initial time step interval.

NOR The number of rays for a given run.

NPLOT The plot number.

NPT If NPT is zero printed output occurs only for the initial and terminal ray points. If NPT is not zero printed output occurs for the first ray point, those points which are an integral multiple of SK, and the last point.

NSH If NSH is zero the shoreline is not drawn on a plot. If NSH is not zero the shoreline is drawn.

NXCMAT If NXCMAT is zero a water depth grid is read in the input for the run. If NXCMAT is not zero the depth grid for the previous run is used again.

n An index to number ray points.

PATI (PSAV) A program symbol for p. The value of p at the point prior to the new ray point.

PCTDIF An estimate of how well the quadratic surface fits the 12 water depths used to derive it. At each of the 4 water depths closest to the ray point, the percentage difference between the water depth derived from the surface fit and the actual depth is computed. PCTDIF is the maximum of these differences.

PHI	The program symbol for ϕ .
POT	(SPOT) A program symbol for p. The value of p at the new ray point.
PREV	(SPREV) The value of v at the previous ray point.
PROJECT	A label of 6 letters and numbers used to identify a computer run.
P(i)	Program symbols for p. The values of p at points intermediate to the new and previous ray points where $i = 1, 2, \dots, 5$.
p	A coefficient of the ray separation equation.
QATI	A program symbol for q. The value of q at the point prior to the new ray point.
QOT	(SQOT) A program symbol for q. The value of q at the new ray point.
Q(i)	Program symbols for q. The values of q at points intermediate to the new and previous ray points where $i = 1, 2, \dots, 5$.
q	A coefficient of the ray separation equation.
RCOUNT	An index to count the number of reflections at a ray point.
REF	The value of REF is determined in MOVE and it denotes the kind of reflection. When there is reflection due to Snell's law with phase velocity REF is 1. When reflection occurs because the packet curvature iteration is not converging REF is 2. If there is reflection because the ray point is too near a reflection point REF is 3.
REFLECT	In MAIN, REFLCT is set equal to zero at the beginning of a ray. In MOVE, REFLCT is set equal to one for those ray points where the conditions for being close to a reflection point are met.
REFRCT	In MAIN, REFRCT is set equal to zero at the beginning of a ray. In HEIGHT, REFRCT is set equal to one for those ray points where there is a breakup of the time step interval due to insufficient accuracy in the Runge-Kutta calculations of β and $d\beta/dt$.

RFLBUM	In MAIN, RFLBUM is set equal to zero at the beginning of a ray. In MOVE, RFLBUM is set equal to one for those ray points where the conditions for being close to a reflection point are met. The value of RFLBUM is used to determine which format statement to use in the output of the ray particulars.
RFRBUM	In MAIN, RFRBUM is set equal to zero at the beginning of a ray. In HEIGHT, RFRBUM is set equal to one for those ray points where there is a breakup of the time step interval due to insufficient accuracy in the Runge-Kutta calculations of β and $d\beta/dt$. The value of RFRBUM is used to determine which format statement to use in the output of the ray particulars.
RHO	The program symbol for ρ .
ROP	The initial value of ROP is determined in the input data. If ROP is zero a ray is not continued beyond a reflection point. If ROP is not zero a ray is continued beyond a reflection point. After a reflection ROP is set equal to zero so that a ray is not continued beyond a second reflection point if one should exist.
RT	The length of the x-axis in inches or centimeters for a given plot.
S	A two dimensional array of numbers used in computing the array E.
SCL	The scale of the plot.
SCLI	The reciprocal of SCL.
SIGMA	The program symbol for σ .
SK	See NPT.
SSALFA	The program symbol for the value of α at the previous ray point.
s_G	The arc length of a wave packet trajectory (ray).
s_v	The arc length of a monochromatic ray.
T	The wave period.

TIME	(TIMEQ) The travel time along a ray.
TPI	The initial value of the wave packet direction used in the analytical solutions of β and $d\beta/dt$.
TT	The program symbol for T.
t	Time.
U	(SAVU, PU) The collinear group speed $d\omega/dk$.
u_m	The maximum velocity of the fluid at the bottom.
V	(SAVV, PV) The program symbol for v.
VX	The program symbol for $\partial v/\partial x$.
VY	The program symbol for $\partial v/\partial y$.
v	The phase speed of a monochromatic wave.
W	An expression used to relate the first spatial derivatives of v and h.
WBCOP	If WBCOP is zero no test is made to determine if the wave breaks. If WBCOP is not zero a test is made in HEIGHT to determine if the wave breaks.
WL	The program symbol for the deep water value of λ .
X	(SVX, PX) The program symbol for x.
XX	(XS) The program symbol for x at the new ray point.
x	A Cartesian coordinate of the water depth grid.
x'	A Cartesian coordinate in a system chosen such that $\partial h/\partial y' = 0$.
Y	(SVY, PY) The program symbol for y. In the theory an expression used to relate the second spatial derivatives of v to the spatial derivatives of h.
YVW	A one dimensional array used in computing the array E.
YY	(YS) The program symbol for y at the new ray point.

y	A Cartesian coordinate of the water depth grid.
y'	A Cartesian coordinate in a system chosen such that $\partial h / \partial y' = 0$.
α	The angle the x' -axis is rotated with respect to the x -axis such that $\partial h / \partial y' = 0$.
β	The ray separation factor.
γ	The wavelet direction defined with respect to the positive x -axis.
γ'	The wavelet direction defined with respect to the positive x' -axis.
γ^*	A quantity used in calculating the wavelet direction using Snell's law with phase velocity.
Δt	The time step interval between ray points.
ϵ_β	The difference between the fourth and fifth order Runge-Kutta solutions of β .
$\epsilon_{\beta t}$	The difference between the fourth and fifth order Runge-Kutta solutions of $d\beta/dt$.
θ	The wave packet (ray) direction defined with respect to the positive x -axis.
θ'	The wave packet (ray) direction defined with respect to the positive x' -axis.
κ_G	The ray curvature of the wave packet.
κ_v	The ray curvature of a monochromatic wave.
κ_G'	The same as κ_G .
λ	The wavelength
π	3.1415927
ρ	An expression used in the spatial derivatives of G .
ρ_f	The density of the fluid.
σ	An expression used in the spatial derivatives of G .

τ	The tangential stress per unit area at the bottom.
ϕ	The angle $(\theta - \gamma)$.
ϕ'	The same as ϕ .
ω	The radian frequency $(2\pi f)$ of the wave.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) <p>The theory and numerical methods are presented for determining the paths of gravity water wave packets. A ray curvature expression is used to determine the wave packet trajectories where the speed of the packet is given by $G = (d\omega/dk) \cos \phi$. The symbol ω denotes the angular frequency, k is the wave number, and ϕ is the difference between the direction of the wave packet and the direction of the wavelets within the packet. At each point of the</p>		

BLOCK 20

Cont → wave packet trajectory the wavelet direction is determined using Snell's law with phase velocity. The wave height is computed along the wave packet paths accounting for the effects of shoaling, refraction, and energy dissipation. The computer program is described and sample printouts and plots are presented. ↗